

Effects of heat generation and thermal radiation on unsteady three-dimensional electro-magnetohydrodynamic water-based hybrid nanofluid flow containing copper and alumina nanoparticles

K. Rajasekaran and P. Suriyakumar

Department of Mathematics
KPR Institute of Engineering and Technology
Coimbatore-641 407, Tamil Nadu, India
Email: rajaks.maths@gmail.com, suriyakumar.p@kpriet.ac.in

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Abstract

This study investigates the unsteady three-dimensional electro- magnetohydrodynamic (EMHD) flow of a water-based hybrid nanofluid involving copper and alumina nanoparticles under the influence of heat generation and thermal radiation effects. Similarity transformations are used for converting the conservation equations of mass, momentum and energy into a system of nonlinear ODEs. The bvp4c solver is used to solve these equations numerically. The outcome shows the skin friction coefficients and reduced Nusselt number which is determined by various values of parameters. The combined influence of magnetic and electric fields through Lorentz force notably modifies the flow and temperature profiles. Moreover, the flow behavior is notably affected by parameters including the magnetic field, electric field, unsteady parameter, thermal radiation, heat generation, stretching rate and

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Prandtl number. The findings are applicable to cooling systems, rotating machinery, electromagnetic pumps, nuclear reactors and thermal management in microelectromechanical systems (MEMS) where enhanced heat dissipation is crucial.

Nomenclature

a, b, c	positive constants
u, v, w	velocities of x, y and z directions
f, g	dimensionless velocities
C_p	specific heat capacity
T	energy of the fluid
T_w	surface temperature
T_∞	free stream energy
r, s	thermal indices
A	unsteady parameter
M	magnetic field
E_x, E_y	electric field with x and y direction
R_d	thermal radiation
H_s	heat source
S	stretching rate parameter
k	thermal conductivity
Pr	Prandtl number
C_{fx}, C_{fy}	skin frictions in x and y direction
Nu_x	Nusselt number
Re_x, Re_y	Reynolds number

Greek words

η	similarity variable
ρ	density
μ	dynamic viscosity
ν	kinematic viscosity
σ	electrical conductivity
θ	dimensionless temperature
φ_1	Cu nanoparticle
φ_2	Al_2O_3 nanoparticle
τ_w	surface shear stress

Sub scripts

- w conditions on wall
- ∞ free stream
- 1 (Cu)
- 2 (Al_2O_3)

1 Introduction

Nanofluids correspond to a novel kind of advanced heat transfer materials developed by uniformly dispersing nanosized particles in typical base fluids include oil, water and ethylene glycol. The thermophysical properties of the base fluid is greatly superior by these nanoparticles, which are usually made of carbides, metals, metal oxides, nitrides, or carbon-based compounds. Choi and Eastman [9] investigated the concept of nanofluids, to address the limitations of traditional coolants which generally exhibit low thermal conductivity. Owing to their superior heat transfer performance, nanofluids are now applied across a wide range of industrial and technological areas, including electronics cooling, nuclear reactors, automotive systems, renewable energy devices and biomedical engineering. Bhattacharya et al. [8] explained laminar required convection of an alumina/water nanofluid in a rectangular silicon microchannel heat sink using a conjugate heat transfer model incorporating thermal dispersion effects. Lin et al. [12] investigated the natural convection and flow of water and aluminium oxide nanofluids in a cavity, highlighting the influence of Prandtl number, Grashof number, volume fraction and nanoparticle diameter on overall augmentation of heat transfer. The heat transfer behavior of MHD 3D nanofluid flow over a bidirectional aggressively extending plane were investigated by Ahmad et al. [3]. Magnetite (Fe_3O_4) nanoparticles have been detected in water-based working fluid and it is predicted that surface temperature will fluctuate exponentially. Nayak et al. [17] examined how thermal radiation and a transverse magnetic field influenced a nanofluid's three-dimensional free convective flow over a sheet that was extending linearly. The unpredictable three-dimensional free convection flow at the point of stagnation over a curved isothermal surface in a nanofluid be examined by Saidin et al. [20]. The analysis focuses on understanding the heat and mass transmission characteristics are affected by surface curvature, unsteadiness and nanoparticle concentration.

Although nanofluids exhibit remarkable heat transfer enhancement, the use of a single type of nanoparticle often restricts the achievable level of thermal improvement. To overcome this limitation, hybrid nanofluids were introduced which is

formed by spreading two different nanoparticles within a single base liquid. These hybrid mixtures exhibit enhanced thermal conductivity, improved stability and superior energy transfer efficiency compared to conventional nanofluids, making them highly suitable for advanced energy, electronic and biomedical applications. Suresh et al. [22] developed hybrid nanofluid composed of copper-alumina/water and demonstrated a notable increase in thermal performance relative to a particle nanofluids. Takabi et al. [23] investigated to using a mixed nanofluid to enhance heat transfer efficiency in a sinusoidal corrugated enclosure. The study utilized thermophysical property values of the hybrid nanofluid. Shah et al. [21] modeled a hybrid nanofluid's natural convection ($Fe_3O_4 - CMWCNT/H_2O$) through a porous medium using the control volume finite element method based on a Non-Darcy flow model incorporating magnetic field effects. Their study emphasized the influence of Hartmann number, permeability, porosity and thermal radiation on both velocity and temperature distributions. Tlili et al. [24] examined the conventional and hybrid nanofluid's heat behavior demonstrating how particle type and concentration significantly affect heat transport mechanisms. Venkateswarlu et al. [26] investigated the copper and alumina in water based hybrid nanofluid movement on a porous stretched sheet under radiative heat was affected by varying viscosity and viscous dissipation. Surface drag and heat transport characteristics are assessed by numerically solving the governing equations using a shooting scheme and the Runge-Kutta-Fehlberg method after transforming them using similarity variables. The flow of a copper-aluminum oxide-water mixed nanofluid over a stretching or contracting Riga plate was investigated by Khashi'ie et al. [10] using EMHD, radiation and nanoparticle shape effects. Likewise, Krishna et al. [11] investigated the radiative unsteady magnetohydrodynamic flow on a vertically porous, accelerated surface with slip and rotational effects of a Casson-type hybrid nanofluid ($Ag - TiO_2/WEG$) revealing the influence of buoyancy forces, Hartmann number and nanoparticle volume fraction on fluid motion and thermal performance. Acharya et al. [1] performed a hydrothermal behavior analysis of a hybrid nanofluid using spectral quasi-linearization simulation. The study employed thermophysical property values of the hybrid nanofluid as shown in the table. Zainal et al. [28] analyzed flow and heat generation/absorption effects of a hybrid nanofluid on a bidirectional exponentially increasing/reducing sheet using similarity transformations and the bvp4c numerical method in MATLAB. Oke et al. [18] analyzed the 3D Copper-alumina/water based nanofluid flow in hybrid and heat transfer over a extend plate under an inclined magnetic field. Hybrid nanofluids offer exceptional capacity for heat transfer useful for advanced thermal applications. Unyong et al. [25] examined the creation of entropy in $Cu - Fe_3O_4$ /ethylene glycol hy-

brid nanofluid flow across a sheet of stretching, incorporating the consequences of thermal radiation, an magnetic field, elastic deformation and an uneven heat source to improve the understanding of underlying physics. Ahmad et al. [5] analyzed the unstable three-dimensional heat transfer in a hybrid nanofluid including brick-shaped $CeO_2 - ZnO$ nanoparticles under a heat source/sink, revealing enhanced thermal conductivity and improved temperature distribution. They further examined the radiative bidirectional magnetized hybrid nanofluid flow using the Keller-Box method, showing that magnetic and radiation parameters strongly influence boundary layer structure, heat transfer rate and flow stability. Zangoee et al. [30] analyzed the three-dimensional hybrid nanofluid flow influenced by a non-uniform magnetic field over an exponentially stretching or shrinking surface, while Asghar et al. [7] analyzed magnetized hybrid nanofluid flow with mixed convection compressing the effects of velocity slip conditions and heat source/sink. Yasmin et al. [27] analyzed heat and mass transfer features of a hybrid nanofluid that includes diamond and copper nanoparticles flowing an integrated stretching surface in a porous material. The thermophysical attributes such as density, thermal conductivity, specific heat and dynamic viscosity were determined using established hybrid nanofluid correlations to provide an accurate prediction of the flow and heat transfer behavior. Muhammad Raza Shah Naqvi et al. [16] emphasized that hybrid nanofluids with a variety of nanoparticles demonstrate enhanced heat transfer capabilities, making them highly suitable for industrial and renewable energy applications.

In recent years, Electro-magnetohydrodynamics (EMHD) has evolved as an extension of classical magnetohydrodynamics (MHD), enclose the combined effects of electric and magnetic fields on the motion electrically conducting fluids. The interaction of these coupled electromagnetic forces substantially modifies velocity profiles and temperature distributions within the fluids. These effects become particularly significant in micro and nanoscale systems, where conventional magnetic forces alone exert a comparatively weaker influence. Zainal et al. [29] studied EMHD unsteady flow of a hybrid nanofluid solving governing equations and stretching/shrinking sheet over a numerically using MATLAB. Similarly, Ahmad et al. [2] observed the bidirectional convective heat transfer behavior of a water-based hybrid nanofluid containing blade-shaped CdTe and graphite nanoparticles, applying scaling-group transformations along with the Keller-box method for numerical analysis. Amiri et al. [6] examined the electrohydrodynamics hybrid nanofluid effect on transfer the heat and fluid flow in three-dimensional chain microchannels. Enhancing heat exchanger performance through chain microchannel configurations offers improved thermal management for high-performance systems. Mkhathshwa

et al. [15] examined the behavior of an electromagnetized carreau hybrid nanofluid flowing through a Darcy-Forchheimer porous medium under slip boundary conditions, using $Ag - Al_2O_3$ /water as the fluid employed. Similarly, Ramesh et al. [19] analyzed the flow of a $TiO_2 - Fe_3O_4$ /ethylene glycol hybrid nanofluid above an exponentially stretched sheet, combining the impacts of EMHD, radiation and viscous dissipation to evaluate their collective influence on thermal performance and energy efficiency.

In this work, the unsteady 3D electro-magnetohydrodynamic (EMHD) flow of a hybrid nanofluid including $Cu - Al_2O_3/H_2O$ based nanoparticles are investigated in relation to heat sources and thermal radiation. EMHD driven hybrid nanofluids have recently attracted significant attention for their enhanced heat transfer efficiency and controllable flow characteristics under combined electric and magnetic fields. The analysis focuses on the important dimensionless parameters are the magnetic field, electric field, unsteady, thermal radiation, heat source and stretching parameter affect the flow and thermal transport efforts. The skin friction coefficients and reduced Nusselt number are assessed both graphically and numerically. It is clear from the literature described above that no earlier research has examined the linked impacts of unsteady 3D EMHD flow of hybrid nanofluids in the simultaneous existence of heat energy and a heat source. Hence, this study make a novel contribution to hybrid nanofluid research by examining the coupled effect of unsteady three dimensional EMHD flow with thermal radiation and heat source. This investigation provides a novel contribution to the hybrid nanofluid field investigated by examining the combined effects of unsteady 3D EMHD flow, nanoparticle interaction and radiative heat transfer which have received limited attention in earlier works. The findings have significant practical relevance for microfluidic pumps, electromagnetic flow regulation, EMHD-direct hybrid nanofluids improved thermal conductivity and electro kinetic behavior are crucial for enhancing performance and efficiency in medical fields (drug distribution and diagnostic devices), modern electronic cooling devices and energy-efficient renewable technology.

2 Mathematical formulation

A Mathematical model, examine the unsteady 3D Cartesian flow of a laminar, viscous and incompressible hybrid nanofluid made from alumina (Al_2O_3) and copper (Cu) nanoparticles scattered in a fluid that is based on water. By using the suitable similarity transformations, the governing PDEs are converted into their di-

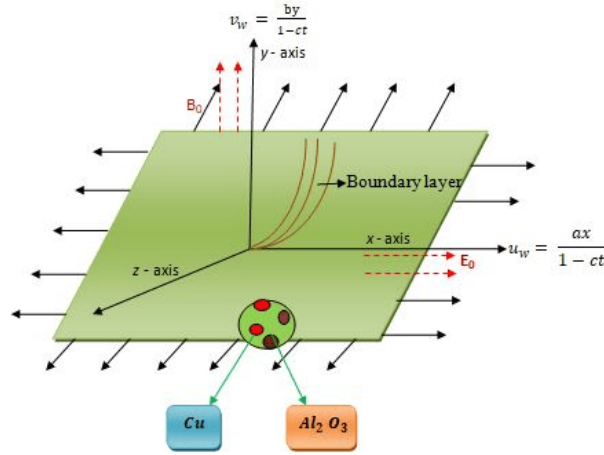


Figure 1: Schematic of the physical model

mensionless form. The velocity profile with x , y and z directions is then revealed by u , v and w , respectively. The unsteadiness in flow arises due to the stretching motion of the surface where the stretching velocities are defined as $u_w = \frac{ax}{1-ct}$ and $v_w = \frac{by}{1-ct}$ with a , b and c are positive constants. In momentum equations, the electro-magnetohydrodynamic (EMHD) body force, called the Lorentz force is included to account for the interaction between the electrically conducting hybrid nanofluid and the applied transverse magnetic and electric fields. The magnetic and electric field is expressed as $B_0 = \frac{b_0}{\sqrt{(1-ct)}}$ and $E_0 = \frac{e_0}{(1-ct)^{3/2}}$ respectively. The heat source parameter is represented as $Q = \frac{Q_0}{(1-ct)}$, where $Q > 0$ corresponds to heat generation. Figure 1 shows the coordinate system and a schematic representation of the physical model.

The physical model is created as the conservation of a structure of connected non-linear partial differential equations of mass, momentum and energy of the hybrid nanofluid by taking advantage of the mentioned variables and using the boundary layer estimation. Under these assumptions, the governing equations are defined as follows: (Ahmad et al. [2] and Nayak et al. [17])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu_{hnf} \frac{\partial^2 u}{\partial z^2} + \frac{\sigma_{hnf}}{\rho_{hnf}} (E_0 B_0 - B_0^2 u) \quad (2.2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu_{hnf} \frac{\partial^2 v}{\partial z^2} + \frac{\sigma_{hnf}}{\rho_{hnf}} (E_0 B_0 - B_0^2 v) \quad (2.3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{hnf} \frac{\partial^2 T}{\partial z^2} + \frac{Q}{(\rho C_p)_{hnf}} (T - T_\infty) - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial q_r}{\partial z} \quad (2.4)$$

The geometrical conditions for present circumstances are,

$$\begin{aligned} u &= \frac{ax}{1-ct}, \quad v = \frac{by}{1-ct}, \quad w = 0 \quad \text{at} \quad z = 0 \\ T &= T_w = T_\infty + \left(\frac{T_0 x^r y^s}{1-ct} \right) \quad \text{at} \quad z = 0 \\ u &\longrightarrow 0, \quad v \longrightarrow 0, \quad T \longrightarrow T_\infty, \quad \text{as} \quad z \longrightarrow \infty. \end{aligned} \quad (2.5)$$

The radiative heat flux q_r in the Rosseland approximation takes the following form: (Mehmood et al. [14] and Nayak et al. [17])

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial z} \quad (2.6)$$

where σ^* denotes Stefan-Boltzmann constant and k^* defines mean absorption coefficient. It is considered flow energy variations are sufficiently minimal to allow for the extension of T^4 in a Taylor series. Therefore, when we expand T^4 around T_∞ and remove higher degree terms, we take,

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (2.7)$$

$$\frac{\partial q_r}{\partial z} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial z^2} \quad (2.8)$$

The set of equations utilized in the above formulation's similarity transformations is expressed as: (Ahmad et al. [2])

$$\begin{aligned} u &= \frac{ax}{1-ct} f'(\eta), \quad v = \frac{ay}{1-ct} g'(\eta), \quad w = -\sqrt{\frac{a\nu_f}{1-ct}} [f(\eta) + g(\eta)], \\ \eta &= \sqrt{\frac{a}{\nu_f(1-ct)}} z, \quad T(x, y, z, t) = T_\infty + [T_w(x, y, t) - T_\infty] \theta(\eta). \end{aligned} \quad (2.9)$$

with the contributions of equations (2.1 - 2.4) and the equations (2.5 - 2.8) using transport equations (2.9) adopt the following forms

$$\frac{\nu_{hnf}}{\nu_f} f''' - f'^2 + f''(f+g) - A \left(f' + \frac{\eta}{2} f'' \right) + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} (ME_x - Mf') = 0 \quad (2.10)$$

$$\frac{\nu_{hnf}}{\nu_f} g''' - g'^2 + g''(f+g) - A \left(g' + \frac{\eta}{2} g'' \right) + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} (ME_y - Mg') = 0 \quad (2.11)$$

$$\left(\frac{\alpha_{hnf}}{\alpha_f} + R_d \left(\frac{(\rho C_p)_f}{(\rho C_p)_{hnf}} \right) \right) \theta'' + Pr \left[\theta'(f+g) - A \left(\theta + \frac{\eta}{2} \theta' \right) - \theta(rf' + sg') + \frac{(\rho C_p)_f}{(\rho C_p)_{hnf}} H_s \theta \right] = 0 \quad (2.12)$$

using boundary conditions

$$\begin{aligned} f(\eta) = 0, g(\eta) = 0, f'(\eta) = 1, g'(\eta) = S, \theta(\eta) = 1 \text{ at } \eta = 0 \\ f'(\eta) \rightarrow 0, g'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty. \end{aligned} \quad (2.13)$$

In the above equation, the magnetic field is denoted by $M = \left(\frac{\sigma_f}{a\rho_f} \right) b_0^2$, while $A = \frac{c}{a}$ defines the unsteady parameter. The ratio of stretching rate parameter $S = \frac{b}{a}$, $E_x = \left(\frac{e_0}{b_0(ax)} \right)$ and $E_y = \left(\frac{e_0}{b_0(ay)} \right)$ describe the electric-field along x and y -direction, $R_d = \frac{16\sigma^* T_\infty^3}{3k^* k_f}$ represents thermal radiation, $H_s = \frac{Q_0}{a(\rho C_p)_f}$ represents the heat source $H_s > 0$ and the Prandtl number is expressed as $Pr = \frac{\nu_f}{\alpha_f}$.

Table 1: Thermophysical properties of H_2O , Cu and Al_2O_3 : [Asghar et al. [7], Zainal et al. [29] and Zangoee et al. [30]].

Properties	H_2O	$Cu(\varphi_1)$	$Al_2O_3(\varphi_2)$
Specific heat [$C_p(J/KgK)$]	4179	385	765
Density of the fluid [$\rho(Kg/m^3)$]	997.1	8933	3970
Electrical conductivity $\sigma(S/m)$	5.5×10^{-6}	5.96×10^7	3.69×10^7
Thermal conductivity [$K(W/mk)$]	0.613	400	40

2.1 Skin friction and Nusselt number

The skin-friction coefficients (C_{fx}, C_{fy}) and reduced Nusselt number are the most notable values for mechanical and thermal processes, as well as the shear stresses along x and y axis is denoted by the symbols τ_{wx} and τ_{wy} respectively, the heat flow is denoted by q_w : (Ahmad et al. [4])

Table 2: Thermophysical properties with hybrid nanofluid values: [Acharya et al. [1], Takabi et al. [23], Yasmin et al. [27], Zainal et al. [29] and Zangoee et al. [30]]

Thermophysical Properties	Hybrid Nanofluid Values
Thermal diffusivity of the hybrid nanofluid	$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C p)_{hnf}}$
Dynamic viscosity of the hybrid nanofluid	$\mu_{hnf} = \frac{\mu_f}{(1-\varphi_1-\varphi_2)^{2.5}}$
Density of the hybrid nanofluid	$\rho_{hnf} = \varphi_1 \rho_1 + \varphi_2 \rho_2 + (1 - \varphi_1 - \varphi_2) \rho_f$
Specific heat capacity of the hybrid nanofluid	$(\rho C p)_{hnf} = \varphi_1 (\rho C p)_1 + \varphi_2 (\rho C p)_2 + (1 - \varphi_1 - \varphi_2) (\rho C p)_f$
Thermal conductivity of the hybrid nanofluid	$\frac{k_{hnf}}{k_f} = \frac{\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi_1 + \varphi_2} + 2k_f + 2(\varphi_1 k_1 + \varphi_2 k_2) - 2(\varphi_1 + \varphi_2)k_f}{\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi_1 + \varphi_2} + 2k_f - (\varphi_1 k_1 + \varphi_2 k_2) + (\varphi_1 + \varphi_2)k_f}$
Electrical conductivity of the hybrid nanofluid	$\frac{\sigma_{hnf}}{\sigma_f} = \frac{\frac{\varphi_1 \sigma_1 + \varphi_2 \sigma_2}{\varphi_1 + \varphi_2} + 2\sigma_f + 2(\varphi_1 \sigma_1 + \varphi_2 \sigma_2) - 2(\varphi_1 + \varphi_2)\sigma_f}{\frac{\varphi_1 \sigma_1 + \varphi_2 \sigma_2}{\varphi_1 + \varphi_2} + 2\sigma_f - (\varphi_1 \sigma_1 + \varphi_2 \sigma_2) + (\varphi_1 + \varphi_2)\sigma_f}$

$$C_{fx} = \frac{\tau_{wx}}{\rho_f u_w^2}, \tau_{wx} = \mu_{hnf} \left(\frac{\partial u}{\partial z} \right)_{z=0} \quad (2.14)$$

$$C_{fy} = \frac{\tau_{wy}}{\rho_f v_w^2}, \tau_{wy} = \mu_{hnf} \left(\frac{\partial v}{\partial z} \right)_{z=0} \quad (2.15)$$

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad q_w = -k_{hnf} \left(\frac{\partial T}{\partial z} \right)_{z=0} + (q_r)_{z=0} \quad (2.16)$$

The dimensionless formulations of the above mentioned quantities using Reynolds's numbers $Re_x = \frac{xu_w}{\nu_f}$, $Re_y = \frac{yu_w}{\nu_f}$. Here, the solutions of skin friction coefficients and reduced Nusselt number as follows:

$$\sqrt{Re_x} C_{fx} = \frac{1}{(1 - \varphi_1 - \varphi_2)^{2.5}} f''(0) \quad (2.17)$$

$$\sqrt{Re_y} C_{fy} = \frac{1}{(S)^{3/2}(1 - \varphi_1 - \varphi_2)^{2.5}} g''(0) \quad (2.18)$$

$$\frac{1}{\sqrt{Re_x}} Nu_x = - \left[\frac{k_{hnf}}{k_f} + R_d \right] \theta'(0) \quad (2.19)$$

3 Numerical methods

A nonlinear boundary value problem is created by combining the boundary values (2.13) with the linked nonlinear ODE equations (2.10 - 2.12). Since the system contains 3^{rd} order derivatives in f''' , g''' and 2^{nd} order derivatives in θ'' , the equations must to be changed into a system of 1^{st} order ODEs in order to find numerical solutions. This transformation facilitates the application of MATLAB bvp4c solver, it was created for efficient boundary value problem solving. The bvp4c method iteratively refines the solution to satisfy both the boundary conditions and the governing equations properly. Finally, the numerical computations are implemented using MATLAB, where the resulting data are presented in the form of tables and graphical illustrations to evaluate the impact of varies physical

parameters over the heat and flow fields.

Table 3: Comparison of the present outcomes with previous investigation in the variations of S and in the absence of $A = M = E_x = E_y = R_d = H_s = \varphi_1 = \varphi_2 = 0$: (Ahmad et al. [2] and Liu and Andersson [13])

	$-f''(0)$			$-g''(0)$		
	Liu and Andersson [13]	Ahmad et al. [2]	Present outcomes	Liu and Andersson [13]	Ahmad et al. [2]	Present outcomes
$S = 0.25$	1.048813	1.048811	1.048814	0.194565	0.194564	0.194565
$S = 0.5$	1.093096	1.093096	1.093095	0.465206	0.465206	0.465206
$S = 0.75$	1.134486	1.134487	1.134486	0.794619	0.794619	0.794619
$S = 1$	1.173721	1.173722	1.173722	1.173721	1.173721	1.173721

To ensure, the dependability of numerical solution for the present problem, a comparative study has been performed as the previously published results of Ahmad et al. [2] and Liu and Andersson [13]. The comparison has been made for the components of $f''(0)$, $g''(0)$ in the absence of $A, M, E_x, E_y, R_d, H_s, \varphi_1$ and φ_2 . The present results display an excellent acceptance with the findings describe in the literature and verify the precision and consistency of the accepted numerical system. The detailed comparison results are obtained in Table 3.

4 Results and discussion

A mathematical model is developed to investigate the effects of radiation, internal heat source and unsteady 3D electro-magnetohydrodynamic (EMHD) convection move of electrically conducting hybrid nanofluid over a plate. The analysis consider the total influence of magnetic and electric fields on the flow behavior. Numerical techniques are employed to obtain the solutions and to study the outcome of different parameters over the fluid movement and heat transfer performance in the governing equation. The key physical parameters involved in this study include the magnetic field (M), unsteady parameter (A), electric field (E_x) in x -direction, electric field (E_y) in y -direction, thermal radiation (R_d), Heat source

(H_s), Stretching parameter (S) and Prandtl number (Pr). To examine the influence of M , A , E_x , E_y , R_d , H_s and S on the velocity components and temperature distribution of the hybrid nanofluids, in addition to the non-dimensional heat transfer rate and C_{fx} , C_{fy} . The parameters are assigned the fixed values: $M = 2.0$, $A = 1.5$, $E_x = 0.3$, $E_y = 0.3$, $R_d = 2.0$, $H_s = 0.2$, $S = 0.5$, $\varphi_1 = 0.01$, $\varphi_2 = 0.01$ and $Pr = 6.2$ (water).

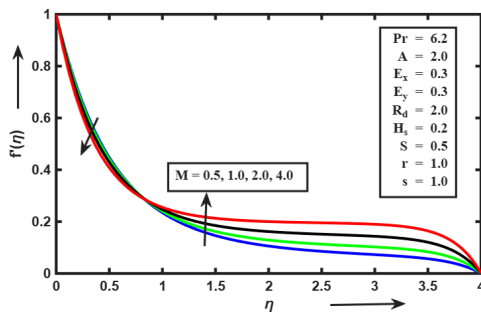


Figure 2: Different values of M on $f'(\eta)$

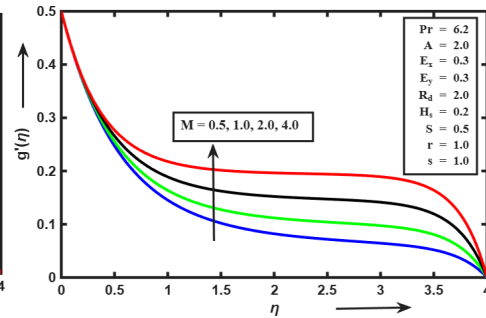


Figure 3: Variations of Magnetic field in $g'(\eta)$

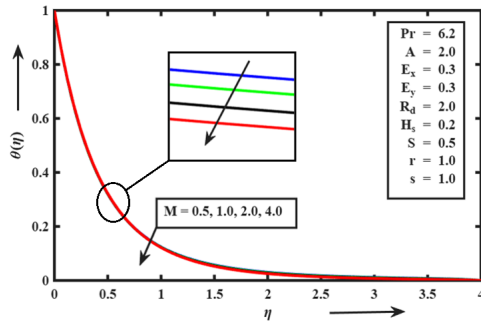


Figure 4: Multiple values of Magnetic field in $\theta(\eta)$

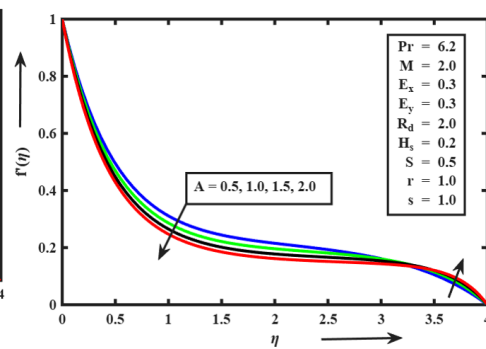


Figure 5: $f'(\eta)$ for various values of A

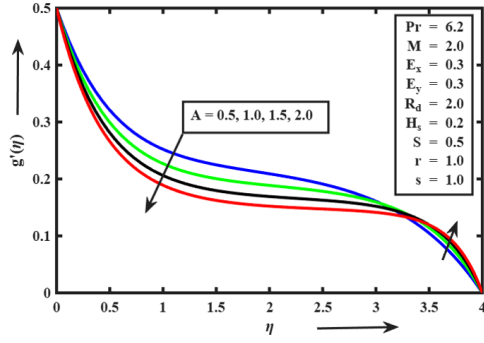


Figure 6: $g'(\eta)$ to multiple values of Unsteady parameter

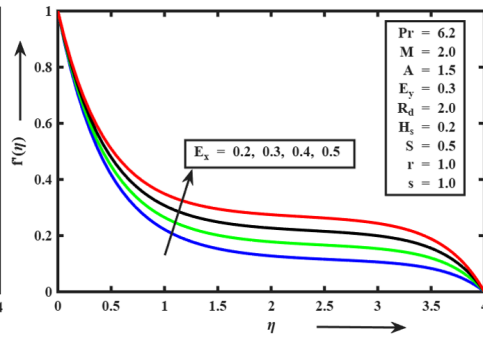


Figure 7: Velocity $f'(\eta)$ with several values of E_x

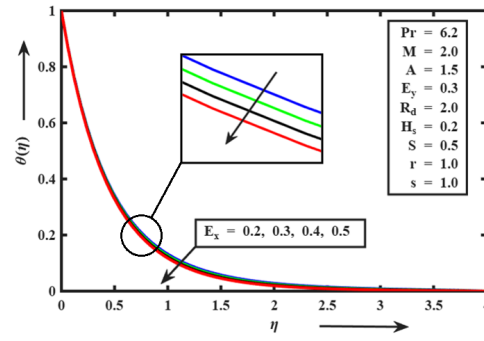


Figure 8: Impact of Electric field E_x on $\theta(\eta)$

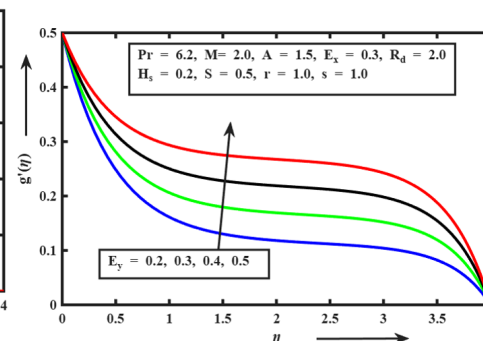


Figure 9: $g'(\eta)$ for several values of E_y

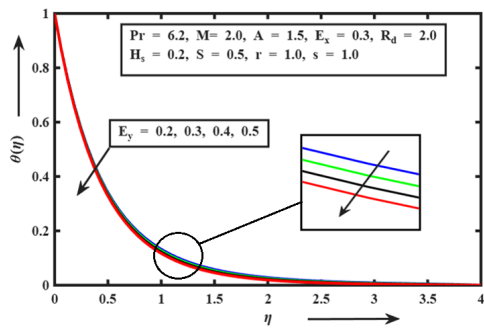


Figure 10: $\theta(\eta)$ for numerous values of E_y

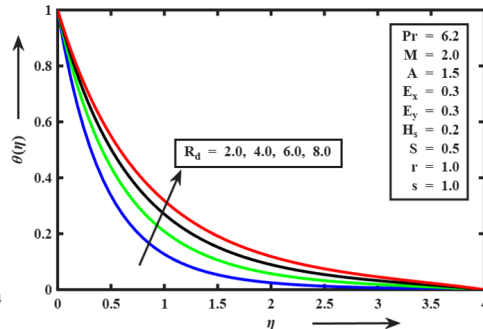


Figure 11: Multiple values of radiation R_d on $\theta(\eta)$

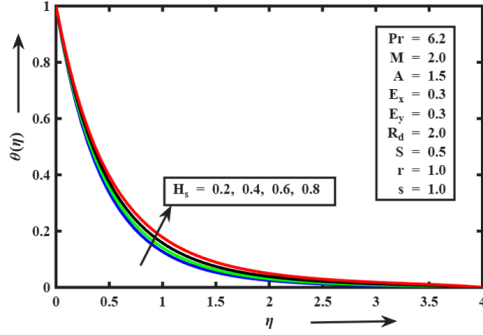


Figure 12: Various values of H_s on temperature profile

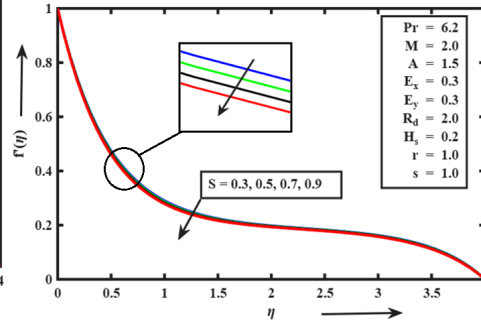


Figure 13: $f'(\eta)$ for variations of S

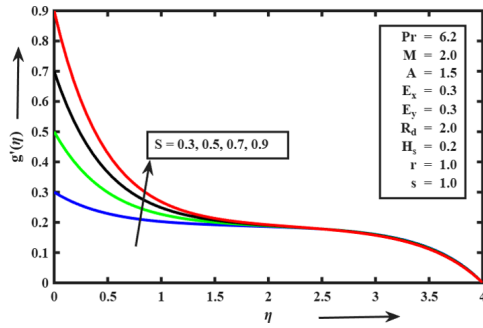


Figure 14: $g'(\eta)$ for several values of S

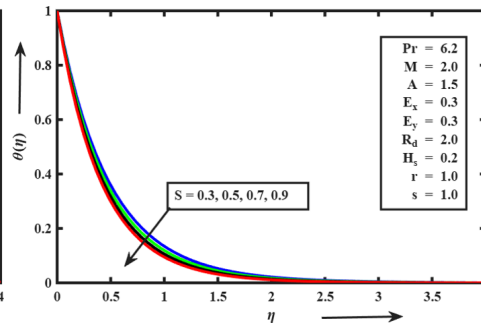


Figure 15: $\theta(\eta)$ for various values of S

This section discusses the outcomes of the parameters on the velocity and temperature profiles as illustrated in figure [2-15].

In Figure 2, expressed the component $f'(\eta)$ decreasing (close to the wall) and enhancing with increasing the magnetic field M . The applied magnetic field make the Lorentz effort opposing the fluid motion, This causes the flow to slow down and reduces the momentum boundary layer thickness. Figure 3, it is noted that the velocity components $g'(\eta)$ increases, also the M enhances. This increase in velocity happens as a result of the applied M creating a Lorentz force that opposes fluid motion, thereby exerting a resistive or drag effect on the flow. As M increases from 0.5 to 4.0, the fluid motion increases and the velocity boundary layers become thicker. Figure 4 shows that the temperature distribution $\theta(\eta)$ reduces with enhancing values of magnetic field (M). The magnetic field converts a part of the fluid's thermal energy into heat energy, which is reduced $\theta(\eta)$ in the boundary layer. Consequently, thermal boundary layer becomes marginally compressed at higher magnetic field strength. From the figures 5 and 6, it is obtained both ve-

locity components $f'(\eta)$ and $g'(\eta)$ decreasing and increasing trend as the unsteady parameter (A) increases. When A varies from 0.5 to 2.0, the fluid flow becomes slower because the unsteadiness, in the system creates resistance to the fluid motion. This resistance reduces the velocity near a surface, which makes the boundary layer thicker. The reduction in both components $f'(\eta)$ and $g'(\eta)$ indicates that higher estimation of A weakens the fluid motion and reduce the momentum transfer within the boundary layer. Hence, as unsteady parameter enhances, the fluid velocity reduces and the boundary layer width become smaller.

Effects of electric field on velocity components and $\theta(\eta)$ are explored in Figures 7-10. From Figure 7, it is observed that increasing the electric field in the x-direction ($E_x = 0.2, 0.3, 0.4, 0.5$) leads to enhances the velocity component $f'(\eta)$. As E_x increases, the fluid near the surface and the velocity component lies faster throughout the boundary layer. In Figure 8, the temperature distribution $\theta(\eta)$ decrease with higher values of E_x , indicating that the thermal effect near the wall is reduced and the boundary layer of heat becomes slightly thinner. In contrast, Figure 9 reveals an increase in the transverse electric field ($E_y = 0.2, 0.3, 0.4, 0.5$) also enhances the velocity component $g'(\eta)$. A higher E_y strengthens the transverse flow, causing $g'(\eta)$ values to rise across the boundary layer. Meanwhile, Figure 10 shows that the temperature distribution $\theta(\eta)$ decreases as electric field E_y enhances, indicating a reduction in thermal energy in the border layer region.

The influence of R_d and H_s on $\theta(\eta)$ are illustrated in Figures 11 and 12. Figure 11 shows that as R_d increases then the temperature distribution $\theta(\eta)$ also enhances, causing a faster temperature profile and a thicker thermal boundary layer. Similarly, Figure 12 investigated that increasing heat source (H_s) values which also enhances the $\theta(\eta)$ and further improved the temperature boundary layer depth. Overall, stronger radiation and greater heat source power result in a more pronounced higher temperature throughout the boundary layer region. Figures 13-15 illustrate the impact of the S for $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$. Figure 13, indicates the $f'(\eta)$ reduces which increases the stretching rate parameter S . This indicates that stronger stretching at the surface resists fluid motion, resulting in the velocity boundary layer being thinner. In contrast, Figure 14 reveals the velocity $g'(\eta)$ enhanced as the stretching parameter S increases. The interaction between the base fluid and nanoparticles enhances momentum transfer, resulting in higher fluid velocity. Moreover, Figure 15 shows that the temperature distributions within the boundary layer are reduced in regard to enhance the S . These proposed that greater expansion defeat temperature variation, leading to a more stable thermal distribution in the hybrid nanofluid. As S represents the ratio of stretching rates along the x and y -directions, an increase in S indicates stronger expansion in the y -direction which

consequently reduces the thermal distribution in the boundary layer.

Table 4: Numerical values for Skin friction coefficient with respect to velocity components f'' and g'' : $M = 2.0$, $A = 1.5$, $R_d = 2.0$, $\varphi_1 = 0.01$, $\varphi_2 = 0.01$, $S = 0.5$, $H_s = 0.2$, $E_x = E_y = 0.3$, $r = s = 1.0$ and $Pr = 6.2$.

M	A	E_x	E_y	S	$\frac{1}{(1-\varphi_1-\varphi_2)^{2.5}} f''(0)$	$\frac{1}{(S)^{3/2}(1-\varphi_1-\varphi_2)^{2.5}} g''(0)$
0.5	1.5	0.3	0.3	0.5	-1.8387	-2.3075
1.0					-1.9105	-2.3115
2.0					-2.0492	-2.3215
4.0					-2.3070	-2.4158
2.0	0.5				-1.6699	-1.6188
	1.0				-1.8066	-1.8818
	1.5				-1.9322	-2.1129
	2.0				-2.0492	-2.3215
		0.1			-2.0371	-2.1108
		0.3			-1.9322	-2.1129
		0.5			-1.8280	-2.1150
		0.7			-1.7243	-2.1171
			0.1		-1.9304	-2.4244
			0.3		-1.9322	-2.1129
			0.5		-1.9340	-1.8038
			0.9		-1.9357	-1.4962
				0.1	-1.7921	-0.6549
				0.2	-1.8066	-1.8818
				0.3	-1.8210	-3.1697
				0.5	-1.8353	-4.5162

Table 5: Numerical values for Nusselt number with respect to temperature distribution θ' : $M = 2.0$, $A = 1.5$, $R_d = 2.0$, $\varphi_1 = 0.01$, $\varphi_2 = 0.01$, $S = 0.5$, $H_s = 0.2$, $E_x = E_y = 0.3$, $r = s = 1.0$ and $Pr = 6.2$.

M	A	E_x	E_y	R_d	H_s	S	$-\left[\frac{k_{hnf}}{k_f} + R_d\right]\theta'(0)$
0.5	1.5	0.3	0.3	2.0	0.2	0.5	7.5131
1.0							7.5097
2.0							7.4981
4.0							7.4690
2.0	0.5						6.0097
	1.0						6.5126
	1.5						7.0133
	2.0						7.4981
		0.1					6.9763
		0.3					7.0133
		0.5					7.0509
		0.7					7.0846
			0.1				6.9750
			0.3				7.0133
			0.5				7.0509
			0.7				7.0874
				2.0			7.0133
				3.0			7.0762
				5.0			8.7983
				7.0			10.2008
					0.2		7.0133
					0.3		6.7283
					0.4		6.4266
					0.5		6.1042
						0.1	6.4094
						0.2	6.8177
						0.3	7.2076
						0.5	7.5814

The numerical results for the C_{fx} , C_{fy} and reduced Nu_x of the hybrid nanofluid are summarized in Tables 4 and 5. Table 4 reports the outcomes of the skin friction coefficients, $f''(0)$ and $g''(0)$ varies parameter results. It has been noted that rising levels of magnetic field (M), unsteady parameter (A) and stretching parameter (S) as decreasing the skin friction coefficients. An enhances the electric field E_x which increase the $f''(0)$ and decrease skin friction coefficient $g''(0)$ of the flow structure within the boundary layer. It is shows that the C_{fx} corresponding to the velocity component $f''(0)$ reduced while the component $g''(0)$ enhanced with

higher values of E_y . This shows that the E_y strengthens the flow resistance and weakens within the boundary layer. Table 5 presents the computed results of the Nusselt number for various parameter values. The rate of heat transmission rises as the values of unsteady parameter (A), electrical fields (E_x, E_y), thermal radiation R_d and stretching parameter (S), whereas it decreases with increasing M and H_s .

5 Conclusion

In this study, the unsteady three-dimensional electro-magnetohydrodynamic flow of a hybrid nanofluid is analyzed by considering the reaction of heat generation and thermal radiation. The hybrid nanofluid incorporate copper and alumina nanoparticles dispersed in a base fluid. The MATLAB bvp4c solver is used to solve the governing PDEs and the associated boundary conditions numerically once they are converted to a system of ODEs using similarity transformations. The consequences of crucial factors like the magnetic field (M), electric fields (E_x, E_y), unsteady parameter (A), thermal radiation (R_d), Heat source (H_s) and stretching rate (S) on velocity and temperature distribution are presented through graphical results. Furthermore, the corresponding values of C_{fx} , C_{fy} and Nu_x of various parameter combinations are obtained and presented the tabular formation. The results of this study are applicable to a number of practical areas such as thermal engineering, biomechanics and medical science. The principal results of the analysis are presented as follows:

- An enhances M and electric field (E_x) which enhances the velocity component $f'(\eta)$ and decreases the $\theta(\eta)$. Furthermore, an increase in M decreases the C_{fy} and the reduced Nusselt number (Nu_x), whereas an increase in E_x decreases the C_{fy} and increases the reduced Nusselt number (Nu_x).
- The velocity component $g'(\eta)$ enhances with increases the magnetic and electric fields in (E_y). Furthermore, increasing M and E_y decreases the skin-friction coefficient C_{fx} . In addition, an increase in E_y increase the reduced Nusselt number (Nu_x), whereas an increase in M decreases the reduced Nusselt number.
- Higher values of R_d and H_s increases the temperature distribution while increasing R_d and decreasing H_s on the Nu_x .
- The velocity component $f'(\eta)$ reduces with higher estimation of the unsteady parameter and stretching parameter. However, the C_{fx} , C_{fy} decreasing and increasing the Nu_x coresspondingly.

Overall, the findings show that fluid flow and heat transfer behaviour in a boundary layer can be effectively tuned by varying the governing parameters. The electro-magnetohydrodynamic (EMHD) hybrid nanofluid demonstrates enhanced heat transfer and improved flow control when electric and magnetic fields are working together. These characteristics make it highly suitable for applications in microfluid systems, electronic cooling systems and biomedical technologies. Hence, EMHD-based hybrid nanofluids offer a promising pathway for efficient thermal management and next-generation energy transfer technologies.

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