

An approach for finding the shortest path in intuitionistic Z- number weighted graphs

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Abstract

In this paper, a modified Dijkstra's algorithm is proposed to address the shortest path problem in an intuitionistic Z-number environment. The proposed algorithm determines the shortest path from a source vertex to a destination vertex and also computes the shortest distance estimate for each vertex from the source.

1 Introduction

Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of fuzzy sets. Nayagam et al [7], Nehi [8] and Rezvani [10] introduced ranking methods for intuitionistic fuzzy numbers.

In 2007, Karunambigai et al [4] refined an approach based on dynamic programming to solve the Shortest Path Problem (SPP) using intuitionistic fuzzy graphs

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(IFGs). In 2010, Gani [2] established a technique to identify intuitionistic fuzzy shortest path (IFSP) for a given network. Jayagowri et al [3] solved the SPP on a network using Trapezoidal Intuitionistic Fuzzy Numbers.

Sari et al. [9] defined intuitionistic Z- numbers (IZNs) by refining the membership and non-membership grades of the constraint and reliability components of Z-numbers. Siddhartha Sankar Biswas [11] has applied the concepts of IFZ-Dijkstra's Algorithm to solve SPP in IFZ Graphs.

2 Preliminaries

Definition 2.1 (Intuitionistic fuzzy number [1])

An intuitionistic fuzzy number A in X is given by $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ define respectively the degree of membership and degree of non-membership of the element $x \in X$ to the set A , which is a subset of X , and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2 (Generalized triangular intuitionistic fuzzy number [5])

A generalized triangular intuitionistic fuzzy number $A = (a_1, a_2, a_3; w, u)$ whose membership and non-membership functions are as follows.

$$\mu_A(x) = \begin{cases} \frac{w(x - a_1)}{a_2 - a_1}, & \text{if } a_1 \leq x < a_2, \\ w, & \text{if } x = a_2, \\ \frac{w(a_3 - x)}{a_3 - a_2}, & \text{if } a_2 < x \leq a_3, \\ 0, & \text{if } x < a_1 \text{ or } x > a_3. \end{cases}$$

$$\nu_A(x) = \begin{cases} \frac{a_2 - x + u(x - a_1)}{a_2 - a_1}, & \text{if } a_1 \leq x < a_2, \\ u, & \text{if } x = a_2, \\ \frac{x - a_2 + u(a_3 - x)}{a_3 - a_2}, & \text{if } a_2 < x \leq a_3, \\ 1, & \text{if } x < a_1 \text{ or } x > a_3. \end{cases}$$

where the values w and u represent the maximum degree of membership and the minimum degree of non-membership respectively, such that they satisfy the

conditions $0 \leq w \leq 1, \quad 0 \leq u \leq 1, \quad 0 \leq w + u \leq 1$.

Definition 2.3 (Formal definition of Z-number [13])

A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in $[0, 1]$.

Definition 2.4 (Intuitionistic Z-number [9])

An intuitionistic Z-number is of the form $Z = (A, B)$, where $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)), x \in X\}$.

Definition 2.5 (Binary operation on Z-numbers [12])

Let (A_1, B_1) and (A_2, B_2) be any two triangular Z-numbers. Then $(A_1, B_1) (+, \min)(A_2, B_2) = (A_1 + A_2, \min(B_1, B_2))$.

Definition 2.6 (Operation on generalized intuitionistic triangular fuzzy numbers [6])

Let $A = (a_1, a_2, a_3, w_1, u_1)$ and $B = (b_1, b_2, b_3, w_2, u_2)$ be two generalized triangular intuitionistic fuzzy numbers. Then

$$A \oplus B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, w_1 + w_2 - w_1 w_2, u_1 u_2).$$

Definition 2.7 (Z-weight of an edge)

A Z-graph is a graph with Z-number weighted edges. The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 2.8 (Z-length of the path)

Consider a path P from a vertex u to vertex v . Suppose it consists of edges $e_1, e_2, \dots, e_n$. Suppose the weights of these edges are the Z-numbers $(A_1, B_1), (A_2, B_2), \dots, (A_n, B_n)$. The Z-length of the path P is denoted by $ZL(P)$ and

$$ZL(P) = (A_1, B_1)(+, \min)(A_2, B_2) \cdots (+, \min)(A_n, B_n).$$

Definition 2.9 (Complement ranking function)

Let $Z = (A, B)$ and R_1 be any ranking function for fuzzy numbers. Let R_2 be a ranking function from F to $[0, 1]$, where F is the set of fuzzy numbers with support contained in $[0, 1]$. Then define

$$R_c(Z) = R_1(A)(1 - R_2(B)).$$

3 Adapting Dijkstra's algorithm to shortest path problem in Z-graphs with intuitionistic environment

Dijkstra's Algorithm is used for finding the shortest path from one vertex to every other vertex within the same graph whose edges have intuitionistic Z-number weights. We use the following method for ordering intuitionistic fuzzy numbers.

3.1 Ranking method for generalized triangular intuitionistic fuzzy number [6]

If $A = (a_1, a_2, a_3, w, u)$ be a generalized triangular intuitionistic fuzzy number, the magnitude of A is defined as follows:

$$\text{Mag}(A) = \frac{1}{12} \left(\frac{4(a_2 - 2(a_1 + a_3)) + 3w(a_1 + a_3)}{w} + \frac{2(a_1 + a_2 + a_3) - 3u(a_2 + a_3)}{1 - u} \right).$$

3.2 Modified Dijkstra's algorithm for IZSPP

We fix suitable ranking functions R_1, R_2 , and comparing two intuitionistic Z-numbers is done based on complement ranking function. The source vertex is denoted by s . P is used to store all currently unvisited vertices of the intuitionistic Z-graph G .

Inputs

$V = \{1, 2, 3, \dots, n\}$ // the set of vertices in the given connected graph with intuitionistic Z-number weighted edges.

s // the source vertex.

$w(i, j) = (A(i, j; w_1, u_1), B(i, j; w_2, u_2))$ // where $w(i, j)$ is the intuitionistic Z-number weight associated with the edge joining vertex i and vertex j .

Initialization

$IZd[s] = ((0, 0, 0; 1, 0), (1, 1, 1; 1, 0))$ // IZ path from source s to s
for every vertex $i \in V, i \neq s$

{
 $IZd[i] = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$ // IZ path from source s to vertex i
 $predecessor[i] = \varphi$
}

$IZdmin = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$

Main Part

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     $P = V \setminus \{s\}$  //unvisited vertex set
     $N = s$ 
    While  $P \neq \emptyset$ 
    {
    For every  $i$  in  $P$  adjacent to  $N$ 
    {
    If  $IZd[i] > IZd[N] (+, \min) w(i, N)$ 
    {
     $IZd[i] = IZd[N] (+, \min) w(i, N)$ 
     $predecessor[i] = N$ ,
    }
    If  $IZd[i] < IZdmin$ 
     $next = i$ ,
     $IZdmin = IZd[i]$ 
    }
     $N = next$ 
    Delete  $N$  from  $P$ 
     $IZdmin = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$ 
     $\delta$  Output  $IZd[i]$  // IZ length of shortest path from source to  $i$ 
     $predecessor[i]$  // SP from source to  $i$  is  $\{\text{SP from source to } predecessor[i]\} \cup \{\text{edge joining } predecessor[i] \text{ and } N\}$ 

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3.3 Example

Consider the IZ-graph G in Figure -1. Here we need to find the shortest path from the source vertex 'A' to the destination vertex 'E.'

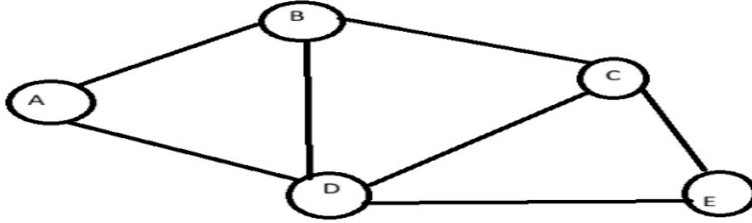


Figure 1: IZ graph G

The edge weights of G are represented as a generalized triangular intuitionistic Z -numbers, which are given in the Table 1.

Table 1: IZ-weight of G

Edge	Intuitionistic Z-number weight
AB	$((50, 60, 70; .9, .1), (.8, .9, 1; .9, .1))$
AD	$((75, 85, 95; 1, 0), (.8, .9, 1; .9, .1))$
BC	$((40, 50, 60; .8, .2), (.9, .95, 1; .9, .1))$
BD	$((10, 15, 20; .9, .1), (.8, .9, 1; .9, .1))$
CE	$((10, 20, 30; 1, 0), (.8, .9, 1; .9, .1))$
CD	$((20, 30, 40; .9, .1), (.9, .95, 1; 1, 0))$
DE	$((20, 25, 30; .9, .1), (.9, .95, 1; 1, 0))$

Let $V = \{A, B, C, D, E\}$

Here, we consider the complement ranking function $R = R_1(1 - R_2)$, where R_1 and R_2 are calculated using the formula given in (3.1).

For adding two generalized triangular intuitionistic Z -numbers, we can use the formula in Definition 2.6 and the binary operations introduced by Stephen [12].

Take $s = A$, $IZd[A] = ((0, 0, 0; 1, 0), (1, 1, 1; 1, 0))$

$IZd[B] = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$

$IZd[C] = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$

$IZd[D] = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$

$IZd[E] = ((\infty, \infty, \infty; 1, 0), (1, 1, 1; 1, 0))$

Then $P = \{B, C, D, E\}$, $N = s = A$.

By applying the modified algorithm, we get the output as

$IZd[A] = ((0, 0, 0; 1, 0), (1, 1, 1; 1, 0))$

Predecessor $[B] = A$ and $IZd[B] = ((50, 60, 70; 1, 0), (.8, .9, 1; .9, .1))$

SP from source to $B = \{A\} \cup \{AB\} = \{AB\}$

Predecessor $[D] = B$ and $IZd[D] = ((60, 75, 90; 1, 0), (.8, .9, 1; .9, .1))$

SP from source to $D = \text{SP from source to } B \cup \{BD\} = \{AB, BD\}$

Predecessor $[C] = D$ and $IZd[C] = ((80, 105, 130; 1, 0), (.8, .9, 1; .9, .1))$

SP from source to $C = \text{SP from source to } D \cup \{CD\} = \{AB, BD, CD\}$

Predecessor $[E] = D$ and $IZd[E] = ((80, 100, 120; 1, 0), (.8, .9, 1; .9, .1))$

SP from source to $E = \text{SP from source to } D \cup \{DE\} = \{AB, BD, DE\}$ Hence, the shortest path from source A to destination E is

$$A \rightarrow B \rightarrow D \rightarrow E$$

and the length of this path is $((80, 100, 120; 1, 0), (.8, .9, 1; .9, .1))$.

4 Conclusion

In this paper, we have made suitable adjustment to the well- known Dijkstra's algorithm, to make its application to a IZ-graph for finding the shortest path from a source vertex to a destination vertex, along with obtaining the shortest length of the path.

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