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Complex valued dislocated metric spaces and fixed point theorem for pair of contractive maps

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Abstract

In this paper, we review, generalize, and establish some fixed-point theorems for contractive mapping in complex-valued dislocated metric space.The obtained results unify and generalize several existing results from the literature [5].

1 Introduction

BCP(Banach contraction principle) [1], which has numerous applications in the fields mathematics, science and engineering, is one of the most crucial relationships in investigating non-linear equations. By utilizing different contractive conditions in an ambient space, numerous extensions and generalizations have been made. These contractive circumstances are crucial for demonstrating the implicitness and exclusiveness of a fixed point.
During their studies in 2000 and 2001, Hitzler and Seda [2]and Hitzler [3] generalized the BCP (Banach contraction principle) [1] in d- metric space. In this space, the distance between two points does not have to be zero.
However, Azam et al. [4], who defined the concept of complex valued metric space

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and demonstrated the Banach contraction principle. Several results have been proved for fixed points in a complex-valued metric space by many researchers. Quickly, Ozgur Ege and Ismet, Karaca [5] delivered the theory of complex valued dislocated metric spaces, and this space was constructed by Bunch, Kannan and Chatterjee to prove certain fixed point theorems

2 Basic Concept and Preliminaries Notes

Here, we begin with some fundamental definitions and notations that will be necessary to understanding and applying our findings [4]. Consider $\Lambda_1, \Lambda_2 \in \mathbb{C}$, where $\mathbb{C}$ be the set of complex numbers. Define a partial order $\preceq$ on $\mathbb{C}$ as follows:

$\Lambda_1 \preceq \Lambda_2$ if and only if $Re(\Lambda_1) \leq Re(\Lambda_2)$ and $Im(\Lambda_1) \leq Im(\Lambda_2)$.

As a result, one can presume that $\Lambda_1 \preceq \Lambda_2$, whenever one of the following situations occurs

(C_1) $Re(\Lambda_1) = Re(\Lambda_2)$ and $Im(\Lambda_1) < Im(\lambda_2)$,

(C_2) $Re(\Lambda_1) < Re(\Lambda_2)$ and $Im(\Lambda_1) = Im(\Lambda_2)$,

(C_3) $Re(\Lambda_1) < Re(\Lambda_2)$ and $Im(\Lambda_1) < Im(z\Lambda_2)$,

(C_4) $Re(\Lambda_1) = Re(\Lambda_2)$ and $Im(\Lambda_1) < Im(\Lambda_2)$.

Specifically, we compose $\Lambda_1 \precnsim \Lambda_2$ if $\Lambda_1 \neq \Lambda_2$ and one of $(C_1), (C_2)$ and (C_3) is satisfied and we write $\Lambda_1 \prec \Lambda_2$ if only (C_3) is satisfied. Notice that:

(1) If $\alpha_1, \alpha_2 \in R$ with $\alpha_1 \leq \alpha_2$, then $\alpha_1 \Lambda \prec \alpha_2 \Lambda$ for all $\Lambda \in \mathbb{C}$.

(2) If $0 \precsim \Lambda_1 \precnsim \Lambda_2$, then $|\Lambda_1| < |\Lambda_2|$.

(3) If $\Lambda_1 \precsim \Lambda_2$ and $\Lambda_2 \prec \Lambda_3$, then $\Lambda_1 \prec \Lambda_3$.

Now, the opinion of complex valued dislocated metric space is given [5].

Definition 2.1. *Suppose $\mu_d : W \times W \to \mathbb{C}$ be a mapping, where μ_d is a non void set satisfies the following conditions:*

(d_1) $\mu_d(a_1, a_2) = \mu_d(a_2, a_1)$;

(d_2) $\mu_d(a_1, a_2) = \mu_d(a_2, a_1)$ *iff* $a_1 = a_2$;

(d_3) $\mu_d(a_1, a_2) \precsim \mu_d(a_1, a_3) + d(a_3, a_2) \; \forall \; a_1, a_2, a_3 \in W$.

Thereafter (W, μ_d) *known as a complex valued dislocated metric space, where* μ_d *kown as a complex valued dislocated metric on* W.

Example 2.1. *Let* $\mu_d : W \times W \to \mathbb{C}$ *be defined by* $\mu_d(\theta_1, \theta_2) = max(\theta_1, \theta_2)$*, then it is called as complex valued dislocated metric space.*

Remark 2.1. *It is true that every complex metric space is also a complex valued dislocated metric space, but the opposite need not be true.*

Definition 2.2. *[5] Given a complex-valued dislocated metric space* (W, μ_d)*, and define a sequence* $\theta_n \in W$ *because* $\theta \in W$.

1. *Consider the sequence* $\{\theta_n\}$ *be convergent to* θ *in* (W, μ_d) *is called complex valued dislocated convergent then for each* $\epsilon > 0 \, \exists \, n_0 \in \mathbb{N}$ *such that* $\mu_d(\theta_n, \theta) < \epsilon$*, for each* $n > n_0$*, which is denoted by* $\theta_n \to \theta$ *as* $n \to \infty$.
2. *Let the sequence* $\{\theta_n\}$ *be Cauchy sequence in complex valued dislocated metric space* (W, μ_d) *If* $\lim_{n\to\infty} \mu_d(\theta_n, \theta_{n+p}) = 0$.
3. *If each Cauchy sequence in* W *converges to a particular* $\theta \in W$*, in which case* (W, μ_d) *is a complex valued complete dislocated metric space.*

Now, to support our main results, we state the two lemmas that are relevant.

Lemma 2.1. *Let* $\{\theta_n\}$ *be a sequence on complex valued dislocated metric space* (W, μ_d)*. Then* $\{\theta_n\}$ *converges to* θ *if and only if* $|\mu_d(\theta_n, \theta)| \to 0 \;\; as \;\; n \to \infty$.

Lemma 2.2. *Assume that,* $\{\theta_n\}$ *be a sequence on complex valued metric space* (W, μ_d)*. Then* $\{\theta_n\}$ *is a Cauchy sequence if and only if* $|\mu_d(\theta_n, \theta_{n+p})| \to 0 \;\; as \; n \to \infty$ *where* $p \in \mathbb{N}$.

Definition 2.3. *Consider A mapping* $H : W \to W$ *contraction mapping on complex valued dislocated metric space*(W, μ_d)*, if ther exist* $0 < r < 1$ *such that* $\mu_d(H\theta_1, H\theta_2) \precsim r\mu_d(\theta_1, \theta_2)$*, for* $\theta_1, \theta_2 \in W$.

3 Main Results

Theorem 3.1. *Assume that the two self mappings* $E, F : W \to W$ *on complete complex valued dislocated metric space*(W, μ_d)*, that satisfy the conditions*

$$\mu_d(E\theta_1, F\theta_2) \precsim \lambda\mu_d(\theta_1, \theta_2). \tag{3.1}$$

Then E and F have a unique common fixed point.

Proof. Consider $\theta_0 \in W$, and the sequence $\{\theta_{2i}\}$ defined by

$$\theta_{2i+1} = E\theta_{2i} = E^{2i+1}, \text{ and}$$
$$\theta_{2i+2} = F\theta_{2i+1} = F^{2i+2}.$$

Then from (3.1), we get

$$\begin{aligned}\mu_d(\theta_{2i+1}, \theta_{2i+2}) &= \mu_d(E\theta_{2i}, F\theta_{2i+1}) \\ &\precsim \lambda^4 \mu_d(\theta_{2i-2}, \theta_{2i-1}).\end{aligned}$$

Continuing this process, we have

$$\mu_d(\theta_{2i+1}, \theta_{2i+2}) \precsim \lambda^{2i} \mu_d(\theta_0, \theta_1). \tag{3.2}$$

Thus

$$\mu_d(\theta_{2i+1}, \theta_{2i+2})| \leq \lambda^{2i} |\mu_d(\theta_0, \theta_1)|. \tag{3.3}$$

Let us use definition 2.1 (d_3) for all $i, j \in \mathbb{N}$ with $i < j$,

$$\begin{aligned}\mu_d(\theta_{2i}, \theta_{2j}) &\precsim \mu_d(\theta_{2I}, \theta_{2i+1}) + \mu_d(\theta_{2i+1}, \theta_{2j}) \\ &\precsim \mu_d(\theta_{2i}, \theta_{2i+1}) + \mu_d(\theta_{2i+1}, \theta_{2i+2}) + \cdots + \mu_d(\theta_{2i-1}, \theta_{2j}) \\ &\precsim (\lambda^{2i} + \lambda^{2i+1} + \cdots + \lambda^{2j_1}) \mu_d(\theta_0, \theta_1) \\ &\precsim \lambda^{2i}[1 + \lambda + \lambda^2 + \cdots + \lambda^{i-j-1}] \mu_d(\theta_0, \theta_1) \\ &\precsim \frac{\lambda^{2i} - \lambda^{2j}}{1 - \lambda} \mu_d(\theta_0, \theta_1)\end{aligned}$$

therefore, we get

$$|\mu_d(\theta_{2i}, \theta_{2j})| \leq \frac{\lambda^{2i} - \lambda^{2j}}{1 - \lambda} |\mu_d(\theta_0, \theta_1)|. \tag{3.4}$$

Since $\lambda \in [0, 1)$, taking limit $n \to \infty$. Then

$$\frac{\lambda^{2i} - \lambda^{2j}}{1 - \lambda} |\mu_d(\theta_0, \theta_1)| \to 0., i.e., |\mu_d(\theta_0, \theta_1)| \to 0.$$

Finalize that $\{\theta_{2i}\}$ is complex valued dislocated Cauchy sequence by Lemma 2.2. As a result, θ_{2i} is complex valued and dislocated convergent to v, and there is an element v in W.

$$\begin{aligned} \mu_d(v, Ev) &\precsim \mu_d(v, \theta_i) + \mu_d(\theta_i, Ev) \\ &= \mu_d(v, \theta_i) + \mu_d(E\theta_i, Ev) \\ &\precsim \mu_d(v, \theta_i) + \lambda\mu_d(\theta_i, v). \end{aligned}$$

As a result, we conclude that $\mu_d(v, Ev) = 0$ because $\{\theta_i\}$ is complex valued dislocated convergent to v as $n \to \infty$. By (d_3) we have $Ev = v$. Similarly, we can prove that $Fv = v$. Hence $Ev = v = Fv$. Thus, v is a common fixed point of E and F in W. Now we shall prove that, the unique common fixed point of E and$F$ in W. Suppose $u \neq v$ be another common fixed point of E and F. Now from (3.1), we get

$$\begin{aligned} \mu_d(v, u) &= \mu_d(Eu, Fv) \\ &\precsim \lambda\mu_d(v, u). \end{aligned}$$

Thus, $|\mu_d(v, u)| \leq \lambda|\mu_d(v, u)|. \Rightarrow (1 - \lambda)|\mu_d(v, u)| \leq 0$. Since $\lambda \in [0, 1)$, so, we get $|\mu_d(v, u)| = 0$. Hence $v = u$. i.e., v is unique common fixed point of E and F in W. This completes the proof. □

Theorem 3.2. *Consider $E, F : W \to W$ be a pair mappings on complete complex valued dislocated metric space (W, μ_d), satisfying the conditions for $0 \leq \lambda < 1$*

$$\mu_d(E\theta_1, F\theta_2) \precsim \lambda[\mu_d(\theta_1, E\theta_1) + \mu_d(\theta_2, F\theta_2)] \tag{3.5}$$

for $\theta_1, \theta_2 \in W$. Then E and F have a unique common fixed point in W.

Proof. : Let $\theta_0 \in W$. and the sequence $\{\theta_{2i}\}$ defined by

$$\begin{aligned}&\theta_{2i+1} = E\theta_{2i} = E^{2i+1},\\&and\\&\theta_{2i+2} = F\theta_{2i+1} = F^{2i+2}.\end{aligned}$$

Then from (3.5), we have

$$\begin{aligned}\mu_d(\theta_{2i}, \theta_{2i+1}) &= \mu_d(E\theta_{2i-1}, F\theta_{2i})\\&\precsim \lambda[\mu_d(\theta_{2i-1}, E\theta_{2i-1}) + \mu_d(\theta_{2i}, F\theta_{2i})]\\&= \lambda[\mu_d(\theta_{2i-1}, \theta_{2i}) + \mu_d(\theta_{2i}, \theta_{2i+1})].\end{aligned}$$

Therefore

$$\mu_d(\theta_{2i}, \theta_{2i+1}) \precsim \frac{\lambda}{1-\lambda}\mu_d(\theta_{2i-1}, \theta_{2i}).$$

Implies that

$$\mu_d(\theta_{2i}, \theta_{2i+1}) \precsim h\mu_d(\theta_{2i-1}, \theta_{2i}).$$

If we continue in the same way, we get

$$\begin{aligned}\mu_d(\theta_{2i}, \theta_{2i+1}) &\precsim h\mu_d(\theta_{2i-1}, \theta_{2i})\\&\precsim h^2\mu_d(\theta_{2i-2}, \theta_{2i-1})\\&\vdots\\&\precsim h^{2i}\mu_d(\theta_0, \theta_1).\end{aligned}$$

Thus, we have

$$|\mu_d(\theta_{2i}, \theta_{2i+1})| \leq h^{2i}|\mu_d(\theta_0, \theta_1)|. \tag{3.6}$$

On the other hand, from the triangle inequality,

$$\begin{aligned}\mu_d(\theta_{2i},\theta_{2i+2k}) &\precsim \mu_d(\theta_{2i},\theta_{2i+1})+\mu_d(\theta_{2i+1},\theta_{2i+2}+...+\mu_d(\theta_{2i+2k-1},\theta_{2i+2k})\\ &\precsim (h^{2i}+h^{2i+1}+...+h^{2i+2k+1})\mu_d(\theta_0,\theta_1)\\ &= \frac{h^{2i}}{1-h}\mu_d(\theta_0,\theta_1).\end{aligned}$$

Thus ,we have

$$|\mu_d(\theta_{2i},\theta_{2i+2k})| \leq \frac{h^{2i}}{1-h}|\mu_d(\theta_0,\theta_1)|. \tag{3.7}$$

From the fact that $0 \leq h < 1$, taking limit as $i \to \infty$, then $|\mu_d(\theta_{2i},\theta_{2i+2k})| \to 0$. By Lemma 2.5, $\{\theta_{2i}\}$ is a complex valued dislocated Cauchy sequence. There is a point $v \in W$ Such that

$$\lim_{i\to\infty} \mu_d(\theta_{2i},v) = 0. \tag{3.8}$$

Because of the completeness of (W,μ_d). We need to show that v is a common fixed point of E and F in W. First we prove that v is a fixed point of E. For this purpose, we use (3.5) as follows:

$$\begin{aligned}\mu_d(v,Ev) &\precsim \mu_d(v,\theta_{2i})+\mu_d(\theta_{2i},Ev)\\ &= \mu_d(\theta_{2i})+\mu_d(E\theta_{2i-1},Ev)\\ &\precsim \mu_d(v,\theta_{2i})+\lambda[\mu_d(\theta_{2i-1},\theta_{2i})+\mu_d(v,Ev)]\\ &\precsim \mu_d(v,\theta_{2i})+\lambda\mu_d(v,Ev)+\lambda h^{2i-1}\mu_d(\theta_0,\theta_1)\\ &\precsim \frac{1}{1-\lambda}\mu_d(v,\theta_{2i}+h^{2i}\mu_d(\theta_0,\theta_1).\end{aligned}$$

We obtain $\mu_d(v, Ev) = 0$ for $n \to \infty$. On the other hand,

$$\begin{aligned}\mu_d(Ev, v) &\precsim \mu_d(Ev, \theta_{2i} + \mu_d(\theta_{2i}, v))\\ &= \mu_d(Ev, E\theta_{2i-1}) + \mu_d(\theta_{2i}, v)\\ &\precsim \lambda[\mu_d(v, Ev) + \mu_d(\theta_{2i-1}, \theta_{2i})] + \mu_d(\theta 2i, v)\\ &\precsim \lambda\mu_d(\theta_{2i-1}, \theta_{2i}) + \mu_d(\theta_{2i}, v).\end{aligned}$$

Since $\mu_d(v, Ev) = 0$. Taking limit as $n \to \infty$. So, $|\mu_d(Ev, v)| = 0$ i.e., $\mu_d(Ev, v) = 0$. As a result, therefore $\mu_d(v, Ev) = \mu_d(Ev, v) = 0$. Implies that, $Ev = v$. Thus, v is a fixed point of E. Similarly, we can prove that v is fixed point of F such that $Fv = v$ is a fixed point of F. Since $Ev = v$ and $Fv = v$. So, $Ev = v = Fv$. Thus v is common fixed point of E and F in W. Now we show the uniqueness: Let u, v be any two diferent common fixed point of E and F in W. From (3.5), we get

$$\begin{aligned}\mu_d(u, v) &= \mu_d(Eu, Fv)\\ &\precsim \lambda[\mu_d(u, Eu) + \mu_d(v, Fv)]\\ &\precsim \lambda[mu_d(u, u) + \mu_d(v, v)]\\ &= 0.\end{aligned}$$

Thus $|\mu_d(u, v)| = 0$ implies that, $\mu_d(u, v) = 0$. So, $u = v$. Thus, v is unique common fixed point of E and F in W. This completes the proof. □

Theorem 3.3. *Consider $E, F : W \to W$ be a pair mappings on complete complex valued dislocated metric space(W, μ_d), satisfying the conditions for $0 \leq \lambda < 1$*

$$\mu_d(E\theta_1, F\theta_2) \precsim \lambda[\mu_d(\theta_1, F\theta_2) + \mu_d(\theta_2, E\theta_1)] \tag{3.9}$$

for $\theta_1, \theta_2 \in W$. Then E and F have a unique common fixed point in W.

Proof. Let $\theta_0 \in W$, and the sequence $\{\theta_{2i}\}$ defined by

$$\begin{aligned}&\theta_{2i+1} = E\theta_{2i} = E^{2i+1},\\ &and\\ &\theta_{2i+2} = F\theta_{2i+1} = F^{2i+2}.\end{aligned}$$

Then from (3.9), we have

$$\begin{aligned}\mu_d(\theta_{2i},\theta_{2i+1}) &= \mu_d(E\theta_{2i+1},F\theta_{2i})\\ &\precsim \lambda[\mu_d(\theta_{2i-1},F\theta_{2i})+\mu_d(\theta_{2i},E\theta_{2i-1})]\\ &= \lambda[\mu_d(\theta_{2i-1},\theta_{2i+1})+\mu_d(\theta_{2i},\theta_{2i})]\\ &\precsim \lambda[\mu_d(\theta_{2i-1},\theta_{2i})+\mu_d(\theta_{2i},\theta_{2i+1}+\mu_d(\theta_{2i},\theta_{2i-1})+\mu_d(\theta_{2i-1},\theta_{2i})]\\ &= \lambda\mu_d(\theta_{2i},\theta_{2i+1})+3\lambda\mu_d(\theta_{2i-1},\theta_{2i})\end{aligned}$$

implies that

$$\mu_d(\theta_{2i},\theta_{2i+1}) \precsim \frac{3\lambda}{1-\lambda}\mu_d(\theta_{2i-1},\theta_{2i}).$$

Therefore

$$\mu_d(\theta_{2i},\theta_{2i+1}) \precsim r\mu_d(\theta_{2i-1},\theta_{2i}), where\, r = \frac{3\lambda}{1-\lambda}.$$

Applying this procedure consequently, we get

$$\begin{aligned}\mu_d(\theta_{2i},\theta_{2i+1}) &\precsim r\mu_d(\theta_{2i-1},\theta_{2i})\\ &\precsim r^2\mu_d(\theta_{2i-2},\theta_{2i-1})\\ &\vdots\\ &\precsim r^{2i}\mu_d(\theta_0,\theta_1).\end{aligned}$$

Thus we have

$$|\mu_d(\theta_{2i},\theta_{2i+1})| \precsim r^{2i}|\mu_d(\theta_0,\theta_1)|.$$

So, for $i < j$. By triangle inequality

$$\mu_d(\theta_{2i}, \theta_{2i+2j}) \precsim \mu_d(\theta_{2i}, \theta_{2i+1}) + \mu_d(\theta_{2i+1}, \theta_{2i+2}) + \cdots + \mu_d(\theta_{2i+2j-1}, \theta_{2i+2j})$$

$$\begin{aligned} &\precsim (r^{2i} + r^{2i+1} + \cdots + r^{2i+2j-1})\mu_d(\theta_0, \theta_1) \\ &= \frac{r^{2i}}{1-r}\mu_d(\theta_0, \theta_1). \end{aligned}$$

Thus we have

$$|\mu_d(\theta_{2i}, \theta_{2i+2j})| \leq \frac{r^{2i}}{1-r}|\mu_d(\theta_0, \theta_1)|.$$

Since $r \in [0, 1)$. So, $|\mu_d(\theta_0, \theta_1)| \to 0$ where $n \to \infty$ i.e., θ_{2i} is a complex valued dislocated Cauchy sequence. By the completeness of (W, μ_d), there is a point $v \in W$ such that

$$\lim_{i\to\infty} \theta_{2i} = v.$$

Since$E$ and F are continous map. So,

$$\begin{aligned} E(\lim_{i\to\infty} \theta_{2i}) &= \lim E\theta_{2i} \\ &= \lim_{i\to\infty} \theta_{2i+1} \\ &= v. \\ \Rightarrow v = Ev. \end{aligned}$$

Similiarly, we can prove that $v = Fv$. Hence $Ev = v = Fv$. Therefore, v is a common fixed point of E and F in W. Now to prove that the common fixed point of E and F are unique. For this, let v^* be another common fixed point of E and F, that is $Ev^* = v^* = Fv^*$ with $v^* \neq v$. Then we have to show that $v = v^*$. It follows from (3.9) that

$$\begin{aligned}\mu_d(v, v^*) &= \mu_d(Ev, Fv^*)\\ &\precsim \lambda[\mu_d(v, Fv^*) + \mu_d(v^*, Ev)]\\ &= \lambda[\mu_d(v, v^*) + (v^*, v)].\end{aligned}$$

Implies that

$$(1 - 2\lambda)|\mu_d(v, v^*)| = 0.$$

Since $0 < \lambda < 1$. So, $\mu_d(v, v^*) = 0$. Thus, we get $v = v^*$. Hence, v is the unique common fixed point of E and F. This completes the proof. □

Example 3.1. *: Let $W = \mathbb{C}$ be the set of complex number. Define a mapping $\mu_d : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ on a complex valued dislocated metric space$(\mathbb{C}, \mu_d)$ as $z_1 = \alpha_1 + i\beta_1$ and $z_2 = \alpha_2 + i\beta_2$. Now, let $E, F : W \to W$ defined by*

$$E\theta_1 = \{\frac{2\theta_1}{3}\}\, and\, F\theta_2 = \{\frac{4\theta_2}{3}\}\, and\ \mu_d(\theta_1, \theta_2) = \frac{\theta_1}{2} + \frac{\theta_2}{2}. \tag{3.6}$$

So, now

$$\begin{aligned}\mu_d(E\theta_1, F\theta_2) &= \frac{\theta_1}{3} + \frac{2\theta_2}{3}\\ &= \frac{1}{6} + \frac{2}{9}\\ &= \frac{5}{18}.\end{aligned}$$

Now using the contractive condition (3.1) of Theorem 3.1, we have

$$\mu_d(e\theta_1, F\theta_2) \precsim \lambda\, \mu_d(\theta_1, \theta_2)$$

as given that $0 < \lambda < 1$, choose $\lambda = \frac{1}{3}$, then clearly $0 < \lambda < 1$. Now putting

$\theta_1 = \frac{1}{2}$ *and* $\theta_2 = 13$. *Then*

$$\begin{aligned}\frac{5}{18} &\precsim \frac{1}{3}(\frac{\theta_1}{2} + \frac{\theta_2}{3}) \\ &\precsim \frac{1}{3}(\frac{1}{4} + \frac{1}{6}) \\ &\precsim \frac{5}{36}.\end{aligned}$$

Now we use the condition (3.5) of Theorem 3.2, we have

$$\mu_d(E\theta_1, F\theta_2) \precsim \lambda[\mu_d(\theta_1, E\theta_1) + \mu_d(\theta_2, F\theta_2)]. \text{ Then}$$

$$\begin{aligned}\frac{5}{18} &\precsim \frac{1}{3}[(\frac{1}{4} + \frac{1}{6}) + (\frac{1}{6} + \frac{2}{9})] \\ &= \frac{1}{3}[\frac{5}{12} + \frac{5}{18}] \\ &\precsim \frac{25}{108}.\end{aligned}$$

Again we use the condition (3.9) of Theorem 3.3, we have

$$\mu_d(E\theta_1, F\theta_2) \precsim \lambda[\mu_d(\theta_1, F\theta_2) + \mu_d(\theta_2, E\theta_1)]. \text{ Then}$$

$$\begin{aligned}\frac{5}{18} &\precsim \frac{1}{3}[(\frac{1}{4} + \frac{2}{9} + \frac{1}{6} + \frac{1}{6})] \\ &= \frac{1}{3}[\frac{17}{3} + 13] \\ &\precsim 2.\end{aligned}$$

Hence all contractive condition of Theorem 3.1, 3.2 and 3.3 are satisfied and $z = 0$ *is the unique common fixed point of* E *and* F.

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