ISSN: 0304-9787
The Aligarh Bulletin of Mathematics
Volume 41, Number 2 (2022), 117-134

Some soft sets in soft ideal topological spaces

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(Received: November 28, 2022 Accepted: December 31, 2022)

Abstract

In this paper, two types of soft sets in soft ideal topological spaces are introduced and some of their properties are discussed. The concept of soft condense ideals is characterized by these collections of sets. Also, some properties of soft extremally disconnected spaces are investigated. Furthermore, decompositions of some types of soft continuous mappings are given and some equivalent conditions concerning this topic are established here.

1 Introduction and preliminary concepts

The concept of soft sets was first introduced by Molodtsov [9] in 1999 who began to develop the basic of the theory as a new approach for modeling uncertainties. Shabir and Naz [11] used soft set theory on topological spaces and called it soft topological spaces. Also, they introduced the notions of soft interior operator, soft closure operator etc. Kandil et.al [7] introduced the concepts of soft ideals and soft local functions. Yumak and Kaymakci [13] introduced the concepts of soft $\widetilde{\tau}^*$-closed sets and soft regular closed sets in

Keywords and phrases: Soft set; Soft topology; Soft ideal topology.
2020 AMS Subject Classification: 54A05, 54A10.
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soft ideal topological spaces. B. Chen [4] introduced the notion of soft semi open set in soft topological spaces.

The rest of this paper is organized as follows. This section contains some necessary concepts and properties. In Section 2, some results on the soft $R-\tilde{I}$-closed sets and soft $A^*_{\tilde{I}}$-sets are introduced. In Section 3, some properties of soft extremally disconnected spaces are given. In Section 4, we give some decompositions of soft continuity in terms of soft $\alpha-\tilde{I}$-continuity, soft pre $-\tilde{I}$-continuity and soft $A^*_{\tilde{I}}-$ continuity.

Now, we recall some definitions and results which are useful in the sequel.

Definition 1.1. *[9]. Let X be an initial universe, E be a set of parameters, $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set on X, where F is a mapping given by $F:A\to P(X)$.*

Definition 1.2. *[10]. Let (F,A) and (G,B) be two soft sets on the common universe X. Then*
(I) (F,A) is said to be a null soft set, denoted by $\tilde{\phi}$ or ϕ_A, if for all $e\in A$, $F(e)=\phi$ (empty set).
(II) (F,A) is said to be an absolute soft set, denoted by $\tilde{X}$ or X_A, if for all $e\in A$, $F(e)=X$. Clearly, $X_A^c=\phi_A$ and $\phi_A^c=X_A$.
(III). The soft union of (F,A) and (G,B) is the soft set (H,C), where $C=A\cup B$ and for all $e\in C$,

$$H(e)=\begin{cases} F(e), & e\in A\backslash B\\ G(e), & e\in B\backslash A\\ F(e)\cup G(e), & e\in A\cap B\end{cases}$$

(IV) The soft intersection of (F,A) and (G,B) is the soft set (H,C), where $C=A\cap B$ and for all $e\in C$, $H(e)=F(e)\cap G(e)$.
(V) (F,A) is said to be a soft subset of (G,B), denoted by $(F,A)\tilde{\subset}(G,B)$, if $A\subset B$ and $F(e)\subset G(e)$ for every $e\in A$.

In order to efficiently discuss, we consider only soft sets on a universe X with the same set of parameters E. We denote the family of these soft sets by $SS(X)_E$. If (F,E) is a soft set on a universe X with the set of parameters E, we write $(F,E)\in SS(X)_E$.

Definition 1.3. *[1]. The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined as $(F,E)^c = (F^c,E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$, for every $e \in E$.*

Definition 1.4. *[11].*
(I) Let $\tilde{\tau}$ be a collection of soft sets on a universe X with a fixed set of parameters E. Then $\tilde{\tau}$ is called a soft topology on X if the following hold:

(i) $\tilde{X}, \tilde{\phi} \in \tilde{\tau}$;

(ii) If $(F,H),(G,E) \in \tilde{\tau}$, then $(F,H)\tilde{\cap}(G,E) \in \tilde{\tau}$;

(iii) If $\{(F_i,E)\}_{i\in I} \subset \tilde{\tau}$, then $\tilde{\bigcup}_{i\in I}(F_i,E) \in \tilde{\tau}$.

The triple $(X,\tilde{\tau},E)$ is called a soft topological space on X.

(II) Let $(X,\tilde{\tau},E)$ be a soft topological space on X. The members of $\tilde{\tau}$ are called soft open sets and their complements are called soft closed. We denote the set of all soft open (resp. soft closed) sets by $\tilde{S}O(X)_E$ (resp. $\tilde{S}C(X)_E$).
(III) Let $(X,\tilde{\tau},E)$ be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $\tilde{s}Cl(F,E)$, is the soft intersection of all soft closed sets containing (F,E), i.e $\tilde{s}Cl(F,E) = \tilde{\bigcap}\{(H,E) : (H,E) \in \tilde{S}C(X)_E$ and $(F,E)\tilde{\subset}(H,E)\}$. Clearly, $\tilde{s}Cl(F,E)$ is the smallest soft closed set which contains (F,E).

Definition 1.5. *[14]. The soft set $(F,E) \in SS(X)_E$ is called a soft point over X if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E\backslash\{e\}$. In this case the soft point (F,E) is denoted by x_e. The collection of all soft points over X is denoted by $SP(X)_E$.*

Definition 1.6. *[14]. Let $SS(X)_{E_1}$ and $SS(Y)_{E_2}$ be two families of soft sets. Suppose $u : X \to Y$ and $p : E_1 \to E_2$ are two mappings. The soft function $f_{up} : SS(X)_{E_1} \to SS(Y)_{E_2}$ is defined as follows:*

(1) If $(F,E_1) \in SS(X)_{E_1}$, then the image of (F,E_1) under f_{up}, written as $f_{up}(F,E_1) = (f_{up}(F), p(A))$, is a soft set in $SS(Y)_{E_2}$ such that for all $b \in B$,

$$f_{up}(F)(b) = \begin{cases} \bigcup_{a\in p^{-1}(b)\cap A} u(F(a)), & p^{-1}(b)\cap A \neq \phi; \\ \phi, & \text{otherwise.} \end{cases}$$

(2) If $(G, E_2) \in SS(Y)_{E_2}$, then the inverse image of (G, E_2) under f_{up}, written as ${f_{up}}^{-1}(G, E_2) = (f_{up}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_{E_1}$ such that for all $a \in A$,

$$ {f_{up}}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B; \\ \phi, & \text{otherwise}. \end{cases} $$

Definition 1.7. *[14]. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(G, E) \in SS(X)_E$. The soft interior of (G, E) , denoted by $\tilde{s}Int(G, E)$ is the soft union of all soft open sets contained in (G, E) i.e. $\tilde{s}Int(G, E) = \tilde{\bigcup}\{(H, E) : (H, E) \in \tilde{\tau}, (H, E)\tilde{\subset}(G, E)\}$. Clearly $\tilde{s}Int(G, E)$ is the largest soft open set contained in (G, E).*

Definition 1.8. *[7].*
(I)A non-null collection $\tilde{I}$ of soft sets on a universe X with the same set of parameters E is called a soft ideal on X if the following two conditions hold:
(1) If $(F, E), (G, E) \in \tilde{I}$, then $(F, E)\tilde{\cup}(G, E) \in \tilde{I}$,
(2) If $(F, E) \in \tilde{I}$ and $(G, E)\tilde{\subset}(F, E)$, then $(G, E) \in \tilde{I}$,
(II) Let $(X, \tilde{\tau}, E)$ be a soft topological space and $\tilde{I}$ be a soft ideal on X. Then $(X, \tilde{\tau}, E, \tilde{I})$ is called a soft ideal topological space. If $(F, E) \in SS(X)_E$, then the soft operator $* : SS(X)_E \to SS(X)_E$, defined by $(F, E)^*(\tilde{I}, \tilde{\tau})$, where $(F, E)^* = \tilde{\bigcup}\{x_e \in SP(X)_E : (G, E)\tilde{\cap}(F, E) \notin \tilde{I}$ for all $(G, E) \in \tilde{\tau}, x_e\tilde{\in}(G, E)\}$, is the soft local function of (F, E) with respect to $\tilde{I}$ and $\tilde{\tau}$.

Theorem 1.1. *[7]. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. The soft closure operator $\tilde{s}Cl^* : SS(X)_E \to SS(X)_E$, defined by $\tilde{s}Cl(F, E) = (F, E)\tilde{\cup}(F, E)^*$, satisfies Kuratwski's axioms.*

Definition 1.9. *[7]. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $\tilde{s}Cl^* : SS(X)_E \to SS(X)_E$ be the soft closure operator. Then there exists a unique soft topology on X with the same set of parameters E, finer than $\tilde{\tau}$, called the $*$-soft topology, denoted by $\tilde{\tau}^*(\tilde{I})$ or $\tilde{\tau}^*$, given by $\tilde{\tau}^*(\tilde{I}) = \{(F, E) \in SS(X)_E : \tilde{s}Cl^*(F, E)^c = (F, E)^c\}$.*

Definition 1.10. *[12]. Let $(X, \tilde{\tau}, E)$ be a soft topological space. A soft set (F, E) is called soft regular open (resp. soft regular closed) set on X if $(F, E) = \tilde{s}Int\tilde{s}Cl(F, E))$ (resp. $(F, E) = \tilde{s}Cl(\tilde{s}Int(F, E))$. The family of* *all soft regular open (resp. soft regular closed) sets is denoted by* $\tilde{S}RO(X)_E$ *(resp.* $\tilde{S}RC(X)_E$*).*

Definition 1.11. *[8]. Let* $(X, \tilde{\tau}, E)$ *be a soft topological space. A soft set* (F, E) *is called soft locally closed if* $(F, E) = (U, E)\tilde{\cap}(V, E)$ *where* (U, E) *is soft open and* (V, E) *is soft closed. We denote the family of all soft locally closed sets by* $\tilde{S}LC(X)_E$*.*

Definition 1.12. *[4]. Let* $(X, \tilde{\tau}, E)$ *be a soft topological space. A soft set* (F, E) *is said to be a soft semi open set if there exists a soft open set* (U, E) *such that* $(U, E)\tilde{\subset}(F, E)\tilde{\subset}\tilde{s}Cl(U, E)$*. We denote the family of all soft semi open sets by* $\hat{S}SO(X)_E$*.*

Theorem 1.2. *[4]. A soft subset* (F, E) *in a soft topological space* $(X, \tilde{\tau}, E)$ *is soft semi open (resp. soft semi closed) set if and only if* $(F, E)\tilde{\subset}\tilde{s}Cl(\tilde{s}Int(F, E))$ *(resp.* $\tilde{s}Int(\tilde{s}Cl(F, E))\tilde{\subset}(F, E)$*).*

Definition 1.13. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space and* $(F, E) \in SS(X)_E$*. Then* (F, E) *is said to be:*

(I) Soft $\alpha - \tilde{I}$*-open [5] if* $(F, E)\tilde{\subset}\tilde{s}Int(\tilde{s}Cl^*(\tilde{s}Int(F, E)))$*. The complement of a soft* $\alpha - \tilde{I}$*-open set is said to be soft* $\alpha - \tilde{I}$*-closed and the family of all soft* $\alpha - \tilde{I}$*-open (resp. soft* $\alpha - \tilde{I}$*-closed) sets in* $(X, \tilde{\tau}, E, \tilde{I})$ *is denoted by* $\tilde{S}\alpha\tilde{I}O(X)_E$ *(resp* $\tilde{S}\alpha\tilde{I}C(X)_E$*).*

(II)Soft pre-$\tilde{I}$*-open [5] if* $(F, E)\tilde{\subset}\tilde{s}Int(\tilde{s}Cl^*(F, E))$ *. The complement of a soft pre-*$\tilde{I}$*-open set is called soft pre-*$\tilde{I}$*-closed and the family of all soft pre-*$\tilde{I}$*-open (resp. soft pre-*$\tilde{I}$*-closed) sets in* $(X, \tilde{\tau}, E, \tilde{I})$ *is denoted by* $\tilde{S}P\tilde{I}O(X)_E$ *(resp.* $\tilde{S}P\tilde{I}C(X)_E$*).*

(III) Soft semi-$\tilde{I}$*-open [5] if* $(F, E)\tilde{\subset}\tilde{s}Cl^*(\tilde{s}Int(F, E))$ *. The complement of a soft semi-*$\tilde{I}$*-open set is called soft semi-*$\tilde{I}$*-closed and the family of all soft semi-*$\tilde{I}$*-open (resp. semi-*$\tilde{I}$*-closed) sets in* $(X, \tilde{\tau}, E, \tilde{I})$ *is denoted by* $\tilde{S}S\tilde{I}O(X)_E$ *(resp.* $\tilde{S}S\tilde{I}C(X)_E$*).*

(V) Soft $*$*-dense in itself [13] if* $(F, E)\tilde{\subset}(F, E)^*$*.*

(VI) Soft $\tilde{\tau}^*$*-closed [13] if* $(F, E)^*\tilde{\subset}(F, E)$*.*

(VII) A soft point x_e *is called a soft pre-*$\tilde{I}$*-closure point of* (F, E) *[5] if* $(F, E)\tilde{\cap}(H, E) \neq \tilde{\phi}$ *for every* $(H, E) \in \tilde{S}P\tilde{I}O(X)_E$ *such that* $x_e\tilde{\in}(H, E)$*.*

The set of all soft pre-$\tilde{I}$-closure points of (F, E) is called the soft pre-$\tilde{I}$-closure of (F, E) and is denoted by $\tilde{s}p\tilde{I}Cl(F, E)$. Consequently, $\tilde{s}p\tilde{I}Cl(F, E) = \tilde{\bigcap}\{(H, E) : (H, E)\tilde{\in}\tilde{s}p\tilde{I}C(X)_E, (F, E)\tilde{\subset}(H, E)\}$.

Definition 1.14. *[13]. A soft subset (F, E) of a soft ideal topological space $(X, \tilde{\tau}, E, \tilde{I})$ is said to be soft regular-$\tilde{I}$-closed if $(F, E) = (\tilde{s}Int(F, E))^*$. The family of all soft regular-$\tilde{I}$-closed sets is denoted by $\tilde{S}R_{\tilde{I}}C(X)_E$.*

Definition 1.15. *[2]. A soft topological space $(X, \tilde{\tau}, E)$ is said to be soft extremally disconnected if the soft closure of every soft open subset of X is soft open.*

Lemma 1.1. *[3]. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. If (F, E) is soft $*$-dense in itself, then $(F, E)^* = \tilde{s}Cl(F, E)^* = \tilde{s}Cl(F, E) = \tilde{s}Cl^*(F, E)$.*

Lemma 1.2. *[3]. If $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, then the following are equivalent.*

(i) $\tilde{X} = \tilde{X}^*$;
(ii) $\tilde{\tau}\tilde{\cap}\tilde{I} = \tilde{\phi}$;
(iii) If $(H, E) \in \tilde{I}$, then $\tilde{s}Int(H, E) = \tilde{\phi}$;
(iv) $(G, E)\tilde{\subset}(G, E)^*$, for every $(G, E) \in \tilde{\tau}$.

Lemma 1.3. *[2]. For any soft topological space $(X, \tilde{\tau}, E)$, the following are equivalent.*

(i) $(X, \tilde{\tau}, E)$ is soft extremally disconnected.
(ii) Every soft regular open subset of X is soft closed.
(iii) Every soft regular closed subset of X is soft open.

Definition 1.16. *[14]. Let $(X, \tilde{\tau}_1, E_1)$ and $(Y, \tilde{\tau}_2, E_2)$ be two soft topological spaces. A soft function $f_{pu} : SS(X)_{E_1} \to SS(Y)_{E_2}$ is called soft continuous if $f_{pu}^{-1}(G, E_2) \in \tilde{\tau}_1 \forall (G, E_2) \in \tilde{\tau}_2$.*

Definition 1.17. *[5]. Let $(X, \tilde{\tau}_1, E_1, \tilde{I})$ be a soft ideal topological space and $(Y, \tilde{\tau}_2, E_2)$ be soft topological space. A soft function $f_{up} : SS(X)_{E_1} \to SS(Y)_{E_2}$ is called a soft pre-$\tilde{I}$-continuous (resp. soft $\alpha - \tilde{I}$-continuous) function if $f_{up}^{-1}(G, E_2) \in \tilde{S}P\tilde{I}O(X)_{E_1} \forall (G, E_2) \in \tilde{\tau}_2$ (resp. $f_{up}^{-1}(G, E_2) \in \tilde{S}\alpha\tilde{I}O(X)_{E_1}$ for every $(G, E_2) \in \tilde{\tau}_2$.*

2 Soft $R-\tilde{I}$-closed and soft $A^*_{\tilde{I}}$-sets

Definition 2.1. *Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E)\in SS(X)_E$. Then (F,E) is said to be soft semi$^*-\tilde{I}$-open if $(F,E)\tilde{\subset}\hat{s}Cl(\tilde{s}Int^*(F,E))$. We denote the family of all soft semi$^*-\tilde{I}$-open sets by $\tilde{S}S^*\tilde{I}O(X)_E$.*

Example 2.1. *Let $(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space, where $X=\{h_1,h_2\}$, $E=\{e_1,e_2\}$, $\tilde{\tau}=\{\tilde{X},\tilde{\phi},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\tilde{X}),(e_2,\{h_2\})\},$ $\{(e_1,\{h_1\}),(e_2,\tilde{X})\}\}$ and $\tilde{I}=\{\tilde{\phi},\{(e_2,\{h_2\})\}\}$. Take $(H,E)=\{(e_1,\{h_1\})\}$. Then $\tilde{s}Cl(\tilde{s}Int^*(H,E))=\tilde{X}$ and so we have that $(H,E)\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(H,E))$. Hence (H,E) is a soft semi$^*-\tilde{I}$-open set.*

Definition 2.2. *Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E)\in SS(X)_E$. Then (F,E) is said to be a soft $f_{\tilde{I}}$-set if $(F,E)\tilde{\subset}(\tilde{s}Int(F,E))^*$.*

Example 2.2. *Let $(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 2.1. Take $(F,E)=\{(e_1,\{h_1\}),(e_2,\tilde{X})\}$. Then $(\tilde{s}Int(F,E))^*=\tilde{X}$ and so, (F,E) is a soft $f_{\tilde{I}}$-set.*

Definition 2.3. *A soft ideal $\tilde{I}$ of a soft ideal topological space $(X,\tilde{\tau},E,\tilde{I})$ is said to be soft co-dense if $\tilde{I}\cap\tilde{\tau}=\tilde{\phi}$.*

Theorem 2.1. *If $(X,\tilde{\tau},E,\tilde{I})$ is a soft ideal topological space and $(F,E)\in SS(X)_E$, then the following are equivalent:*

(i) $\tilde{X}=\tilde{X}^*$.

(ii) $(F,E)\tilde{\subset}(F,E)^*$, for every $(F,E)\in\tilde{\tau}$.

(iii) $(F,E)\tilde{\subset}(F,E)^*$, for every $(F,E)\in\tilde{S}SO(X)_E$.

(iv) $(V,E)=(V,E)^*$, for every soft regular closed set (V,E).

Proof. (i)$\Longrightarrow$(ii). Follows from Lemma 1.2.

(ii)$\Longrightarrow$(iii). Suppose $(F,E)\in\tilde{S}SO(X)_E$. Then there exists a soft open set (H,E) such that $(H,E)\tilde{\subset}(F,E)\tilde{\subset}\tilde{s}Cl(H,E)$. Since (H,E) is a soft open set, $(H,E)\tilde{\subset}(H,E)^*$. By Theorem 1.2, $(F,E)\tilde{\subset}\tilde{s}Cl(H,E)\tilde{\subset}\tilde{s}Cl(H,E)^*=(H,E)^*\tilde{\subset}(F,E)^*$. Hence $(F,E)\tilde{\subset}(F,E)^*$.

(iii)$\Longrightarrow$(iv). If (V,E) is a soft regular closed set, then (V,E) is soft semi open and soft closed. Therefore $(V,E)\tilde{\subset}(V,E)^*$ and $(V,E)^*\tilde{\subset}(V,E)$. Therefore, $(V,E)=(V,E)^*$.

(iv)$\Longrightarrow$(i). Obvious. □

Theorem 2.2. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. If (F, E) is soft $*$-dense in itself, then $\tilde{s}Int(F, E)^c = \tilde{s}Int^*(F, E)^c$.*

Proof. If $(F, E)\tilde{\subset}(F, E)^*$, then $\tilde{s}Cl(F, E) = \tilde{s}Cl^*(F, E)$, by Lemma 1.1. So $(\tilde{s}Cl(F, E))^c = (\tilde{s}Cl^*(F, E))^c$. Therefore, $\tilde{s}Int(F, E)^c = \tilde{s}Int^*(F, E)^c$. □

Corollary 2.1. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space. If $\tilde{I}$ is soft co-dense, then*

(i) $(F, E)^* = \tilde{s}Cl(F, E)^* = \tilde{s}Cl(F, E) = \tilde{s}cl^*(F, E)$ for every $(F, E) \in \tilde{S}SO(X)_E$;

(ii) $\tilde{s}Int(F, E) = \tilde{s}Int^*(F, E)$ for every $(F, E) \in \tilde{S}SC(X)_E$.

Proof. Follows from Theorems 2.1(iii) and 2.2, and Lemma 1.1. □

Lemma 2.1. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. Then (F, E) is soft locally closed if and only if $(F, E) = (U, E)\tilde{\cap}\tilde{s}Cl(F, E)$ for some $(U, E) \in \tilde{\tau}$.*

Proof. If (F, E) is soft locally closed, then $(F, E) = (U, E)\tilde{\cap}(V, E)$, where (U, E) is soft open and (V, E) is soft closed. Hence $(F, E)\tilde{\subset}(U, E)\tilde{\cap}\tilde{s}Cl(F, E)$. Since $(F, E) = (U, E)\tilde{\cap}(V, E)$, $\tilde{s}Cl(F, E) = \tilde{s}Cl[(U, E)\tilde{\cap}(V, E)]\tilde{\subset}\tilde{s}Cl(U, E)\tilde{\cap}\tilde{s}Cl(V, E) = \tilde{s}Cl(U, E)\tilde{\cap}(V, E)$. So we have $\tilde{s}Cl(F, E)\tilde{\subset}\tilde{s}Cl(U, E)\tilde{\cap}(V, E)$. This shows that $\tilde{s}Cl(F, E)\tilde{\subset}(V, E)$ and $(U, E)\tilde{\cap}\tilde{s}Cl(F, E)\tilde{\subset}(U, E)\tilde{\cap}(V, E) = (F, E)$. Hence $(U, E)\tilde{\cap}\tilde{s}Cl(F, E)\tilde{\subset}(F, E)$. Therefore, $(F, E) = (U, E)\tilde{\cap}\tilde{s}Cl(F, E)$. □

Definition 2.4. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. (F, E) is called soft $R-\tilde{I}$-closed if $(F, E) = \tilde{s}Cl(\tilde{s}Int^*(F, E))$. The soft complement of a soft $R-\tilde{I}$-closed set is called soft $R-\tilde{I}$-open. We denote the family of all soft $R-\tilde{I}$-closed (resp. soft $R-\tilde{I}$-open) sets by $\tilde{S}R\tilde{I}C(X)_E$ (resp. $\tilde{S}R\tilde{I}O(X)_E$).*

Example 2.3. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be the soft ideal topological space, where $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, \{(e_1, \{h_2\})\}, \{(e_1, \tilde{X}), (e_2, \{h_2\})\}, \{(e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\}$ and $\tilde{I} = \{\tilde{\phi}, \{(e_1, \{h_2\})\}\}$. Suppose that $(F, E) = \{(e_1, \{h_1\}), (e_2, \tilde{X})\}$. Then (F, E) is soft $R-\tilde{I}$-closed.*

Definition 2.5. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. $(F, E)$is called soft $A^*_{\tilde{I}}$-set if $(F, E) = (U, E)\tilde{\cap}(V, E)$ where (U, E) is a soft open set and (V, E) is soft $R - \tilde{I}$-closed. The family of all soft $A^*_{\tilde{I}}$-sets is denoted by $\tilde{S}A^*_{\tilde{I}}(X)_E$.*

Example 2.4. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be the soft ideal topological space as in Example 2.3. Take $(U, E) = \{(e_1, \tilde{X}), (e_2, \{h_2\})\} \in \tilde{\tau}$ and $(V, E) = \{(e_1, \{h_1\}), (e_2, \tilde{X})\} \in \tilde{S}R\tilde{I}C(X)_E$. Hence $(F, E) = \{(e_1, \{h_1\}), (e_2, \{h_2\})\} = (U, E)\tilde{\cap}(V, E)$ is soft $A^*_{\tilde{I}}$-set.*

Remark 2.1. *Every soft open set is a soft $A^*_{\tilde{I}}$-set and every soft $R-\tilde{I}$-closed set is a soft $A^*_{\tilde{I}}$-set. The converse of these statements is not true, in general, as shown by the following example.*

Example 2.5. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be the soft ideal topological space as in Example 2.3. The soft set $(F, E) = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$ is soft $A^*_{\tilde{I}}$-set but not soft open and $(G, E) = \{(e_1, \{h_2\})\}$ is a soft $A^*_{\tilde{I}}$-set but not soft $R-\tilde{I}$-closed.*

Theorem 2.3. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X)_E$. The following statements are equivalent:*

(i) (F, E) is a soft open set;

(ii) (F, E) is a soft $\alpha - \tilde{I}$-open and a soft $A^*_{\tilde{I}}$-set;

(iii) (F, E) is a soft pre-$\tilde{I}$-open and a soft $A^*_{\tilde{I}}$-set.

Proof. (i)$\Longrightarrow$(ii). Obvious.

(ii) $\Longrightarrow$(iii). Follows from the fact that every soft $\alpha - \tilde{I}$-open set is soft pre-$\tilde{I}$-open.

(iii)$\Longrightarrow$(i). Suppose (F, E) is a soft pre-$\tilde{I}$-open and soft $A^*_{\tilde{I}}$-set. Then $(F, E)\tilde{\subset}\tilde{s}Int(\tilde{s}Cl^*(F, E))$ and $(F, E) = (U, E)\tilde{\cap}(V, E)$, where (U, E) is soft open and (V, E) is soft $R-\tilde{I}$-closed. Now, $(F, E)\tilde{\subset}\tilde{s}Int(\tilde{s}Cl^*[(U, E)\tilde{\cap}(V, E)])$ $\tilde{\subset}\tilde{s}Int[\tilde{s}Cl^*(U, E)\tilde{\cap}\tilde{s}Cl^*(V, E)]$ $\tilde{\subset}\tilde{s}Int[\tilde{s}Cl^*(U, E)\tilde{\cap}\, \tilde{s}Cl^*(\tilde{s}Cl(\tilde{s}Int^*(V, E)))]\tilde{\subset}$ $\tilde{s}Int[\tilde{s}Cl^*(U, E)\tilde{\cap}\tilde{s}Cl(\tilde{s}Int^*(V, E))] = \tilde{s}Int[\tilde{s}Cl^*(U, E)\tilde{\cap}(V, E)] = \tilde{s}Int\tilde{s}Cl^*$ $(U, E)\tilde{\cap}\tilde{s}Int(V, E)$. Since $(F, E)\tilde{\subset}(U, E)$, $(F, E)\tilde{\subset}(U, E)\tilde{\cap}(F, E)\tilde{\subset}$ $(U, E)\tilde{\cap}$ $[\tilde{s}Int(\tilde{s}Cl^*(U, E)\tilde{\cap}\tilde{s}Int(V, E))] = (U, E)\tilde{\cap}\tilde{s}Int(V, E) = \tilde{s}Int[(U, E)\tilde{\cap}(V, E)] =$ $\tilde{s}Int(F, E)$. Therefore (F, E) is a soft open set. □

Theorem 2.4. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space. Then the soft ideal $\tilde{I}$ is soft co-dense if and only if $\tilde{S}A^*_{\tilde{I}}(X)_E\tilde{\cap}\tilde{I} = \tilde{\phi}$.*

Proof. Suppose $\tilde{S}A^*_{\tilde{I}}(X)_E\tilde{\cap}\tilde{I} = \tilde{\phi}$. Since $\tilde{\tau}\tilde{\subset}\tilde{S}A^*_{\tilde{I}}(X)_E$, $\tilde{\tau}\tilde{\cap}\tilde{I} = \tilde{\phi}$ and so $\tilde{I}$ is soft co-dense. Conversely, assume that $(F,E) \in \tilde{S}A^*_{\tilde{I}}(X)_E\tilde{\cap}\tilde{I}$. Then $(F,E) \in \tilde{I}$ which implies that $\tilde{s}Int^*(F,E) = \tilde{\phi}$, by Lemma 1.2. Also $(F,E) = (U,E)\tilde{\cap}(V,E)$, where (U,E) is soft open and (V,E) is soft $R-\tilde{I}$-closed. So$(F,E) = (U,E)\tilde{\cap}\tilde{s}Cl(\tilde{s}Int^*(V,E))\tilde{\subset}\tilde{s}Cl[(U,E)\tilde{\cap}(\tilde{s}Int^*(V,E)) =\tilde{s}Cl(\tilde{s}Int^*$ $[(U,E)\tilde{\cap}(V,E)]) = \tilde{s}Cl(\tilde{s}Int^*(F,E)) = \tilde{s}Cl(\tilde{\phi}) = \tilde{\phi}$. Therefore, we have $\tilde{S}A^*_{\tilde{I}}(X)_E\tilde{\cap}\tilde{I} = \tilde{\phi}$. □

Theorem 2.5. *Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E) \in SS(X)_E$. Then the following statements are true.*

(i) If (F,E) is soft $\tilde{\tau}^*$-open, then $\tilde{s}Cl(F,E)$ is soft $R-\tilde{I}$-closed.

(ii) If (F,E) is a nonempty soft $R-\tilde{I}$-closed set, then $\tilde{s}Int^*(F,E) \neq \tilde{\phi}$.

(iii) If $(X,\tilde{\tau},E)$ is soft discrete, then every soft subset is a soft $R-\tilde{I}$-closed set and hence is a soft $A^*_{\tilde{I}}$-set.

Proof. (i) Clearly, $\tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F,E)))\tilde{\subset}\tilde{s}Cl(F,E)$. Now $(F,E)\tilde{\subset}\tilde{s}Cl(F,E)$ implies that $\tilde{s}int^*(F,E)\tilde{\subset}\tilde{s}Int^*(\tilde{s}Cl(F,E))$ which, by hypothesis, implies that $(F,E)\tilde{\subset}\tilde{s}Int^*(\tilde{s}Cl(F,E))$. Therefore $\tilde{s}Cl(F,E)\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F,E)))$. Hence $\tilde{s}Cl(F,E) = \tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F,E)))$ and so $\tilde{s}Cl(F,E)$ is soft $R-\tilde{I}$-closed.

(ii) The proof is obvious.

(iii) Since, in a discrete soft space, every soft subset is both soft open and soft closed then $\tilde{s}Cl(\tilde{s}Int^*(F,E)) = \tilde{s}Cl(F,E) = (F,E)$. Hence (F,E) is soft $R-\tilde{I}$-closed and hence a soft $A^*_{\tilde{I}}$-set. □

The following example shows that the converses of Theorem 2.5 are not true in general.

Example 2.6. *Let $(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 2.3.*

(i) If $(F,E) = \{(e_2,\{h_2\})\}$, then $\tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F,E))) = \tilde{s}Cl(\tilde{s}Int^*(\tilde{X}))$ $= \tilde{X} = \tilde{s}Cl(F,E)$. So $\tilde{s}Cl(F,E)$ is soft $R-\tilde{I}$-closed. But (F,E) is not soft $\tilde{\tau}^*$-open.

(ii) If $(G,E) = \{(e_1,\{h_1\})\}$, then $\tilde{s}Int^*(G,E) = (G,E) \neq \tilde{\phi}$. Also, $\tilde{s}Cl(\tilde{s}Int^*(G,E)) = \tilde{s}Cl(G,E) = \tilde{X} \neq (G,E)$. Hence (G,E) is not soft $R-\tilde{I}$-closed.

(iii) Consider the soft ideal topological space $(X,\tilde{\tau},E,\tilde{I})$*, where* $X=\{h_1,h_2,h_3,h_4\}$*,* $E=\{e\}$*,* $\tilde{\tau}=\{\tilde{X},\tilde{\phi},\{e,\{h_1\})\},\{(e,\{h_2\})\},\{(e,\{h_1,h_2\})\},\{(e,\{h_1,h_2,h_4\})\},\{(e,\{h_1,h_2,h_3\})\}\}$ *and* $\tilde{I}=\{\tilde{\phi},\{(e,\{h_1\})\},\{(e,\{h_2\})\},\{(e,\{h_1,h_2\})\}\}$*. Hence* $\tilde{S}A^*_{\tilde{I}}(X)_E=SS(X)_E$*, but* $(X,\tilde{\tau},E)$ *is not soft discrete.*

Theorem 2.6. *Let* $(X,\tilde{\tau},E,\tilde{I})$ *be a soft ideal topological space and* $(F,E)\in SS(X)_E$*. Then the following statements are true.*

(i) (F,E) *is soft* $R-\tilde{I}$*-closed if and only if it is both soft* $semi^*-\tilde{I}$*-open and soft pre-*$\tilde{I}$*-closed.*

(ii) If (F,E) *is a soft* $f_{\tilde{I}}$*-set and soft pre-*$\tilde{I}$*-closed, then* (F,E) *is soft* $R-\tilde{I}$*-closed.*

(iii) If (F,E) *is a soft* $f_{\tilde{I}}$*-set, then* $(F,E)^*$ *is a soft* $R-\tilde{I}$*-closed set.*

(iv) If $\tilde{I}$ *is soft co-dense and* (F,E) *is a soft* $A^*_{\tilde{I}}$*-set, then* (F,E) *is a soft* $f_{\tilde{I}}$*-set.*

Proof. (i) If (F,E) is soft $R-\tilde{I}$-closed, then $(F,E)=\tilde{s}Cl(\tilde{s}int^*(F,E))$. So (F,E) is both soft $semi^*-\tilde{I}$-open and soft pre-$\tilde{I}$-closed. The proof of other direction is clear.

(ii) Since (F,E) is soft $f_{\tilde{I}}$-set, $(F,E)\tilde{\subset}(\tilde{s}Int^*(F,E))^*$. So $(F,E)\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(F,E))$. Since (F,E) is soft pre-$\tilde{I}$-closed, $\tilde{s}Cl(\tilde{s}Int^*(F,E))\tilde{\subset}(F,E)$. Hence $(F,E)=\tilde{s}Cl(\tilde{s}Int^*(F,E))$.

(iii) Since (F,E) is a soft $f_{\tilde{I}}$-set, $(F,E)\tilde{\subset}(\tilde{s}Int^*(F,E))^*$. Then $(F,E)^*\tilde{\subset}(\tilde{s}Int^*(F,E))^{**}\tilde{\subset}(\tilde{s}Int^*(F,E))^*\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(F,E))\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(F,E)^*)$, since a soft $f_{\tilde{I}}$-set is soft $*$-dense in itself. Also, $\tilde{s}Cl(\tilde{s}Int^*(F,E)^*)\tilde{\subset}\tilde{s}Cl(F,E)^*=(F,E)^*$. Hence $(F,E)^*=\tilde{s}Cl(\tilde{s}Int^*(F,E)^*)$ and so $(F,E)^*$ is a soft $R-\tilde{I}$-closed set.

(iv) Since (F,E) is a soft $A^*_{\tilde{I}}$-set, $(F,E)=(U,E)\tilde{\cap}(V,E)$ where (U,E) is soft open and (V,E) is soft $R-\tilde{I}$-closed. Now, $(F,E)=(U,E)\tilde{\cap}(V,E)=(U,E)\tilde{\cap}\,\tilde{s}Cl(\tilde{s}Int^*(V,E))\tilde{\subset}\tilde{s}Cl[(U,E)\tilde{\cap}\tilde{s}Int^*(V,E)]=\tilde{s}Cl(\tilde{s}Int^*[(U,E)\tilde{\cap}(V,E)])=\tilde{s}Cl(\tilde{s}Int^*(F,E))=(\tilde{s}Int^*(F,E))^*$, by Corollary 2.1. Therefore, (F,E) is a soft $f_{\tilde{I}}$-set. □

The following example shows that the converse of Theorem 2.6 (ii) need not be true in general.

Example 2.7. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space as in Example 2.3 and* $(F, E) = \{(e_1, \tilde{X}), (e_2, \{h_2\})\}$. *Then* (F, E) *is soft* $R - \tilde{I}$*-closed, but* $(F, E) \not\subseteq (\tilde{s}Int^*(F, E))^* = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$ *and hence* (F, E) *is not a soft* $f_{\tilde{I}}$*-set.*

Theorem 2.7. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space. Then the soft ideal* $\tilde{I}$ *is soft co-dense if and only if* $\tilde{S}R\tilde{I}C(X, \tilde{\tau}, E) = \tilde{S}R_{\tilde{I}}C(X, \tilde{\tau}^*, E)$.

Proof. Suppose that $\tilde{S}R\tilde{I}C(X, \tilde{\tau}, E) = \tilde{S}R_{\tilde{I}}C(X, \tilde{\tau}^*, E)$. Since $\tilde{X}$ is soft $R - \tilde{I}$-closed, $\tilde{X} \in \tilde{S}R_{\tilde{I}}C(X, \tilde{\tau}^*, E)$. Therefore $(\tilde{s}Int^*(\tilde{X}))^* = \tilde{X}$ which implies that $\tilde{X}^* = \tilde{X}$. Thus $\tilde{I}$ is soft co-dense. The converse follows from Corollary 2.1.

Corollary 2.2. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space. If* $\tilde{I}$ *is soft co-dense, then we can obtain* $\tilde{S}R\tilde{I}C(X, \tilde{\tau}, E) = \tilde{S}R_{\tilde{I}}C(X, \tilde{\tau}^*, E) = \tilde{S}RC(X, \tilde{\tau}^*, E) = \tilde{S}R\tilde{I}C(X, \tilde{\tau}^*, E)$

Proof. Follows from Theorem 2.7 and Corollary 2.1. □

Theorem 2.8. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space. Then* $\tilde{S}R\tilde{I}C(X)_E = \{\tilde{s}Cl(U, E) : (U, E) \in \tilde{S}S^*\tilde{I}O(X)_E\}$.

Proof. If $(F, E) \in \tilde{S}R\tilde{I}C(X)_E$, $(F, E) = \tilde{s}Cl(\tilde{s}Int^*(F, E))$. If $(U, E) = \tilde{s}Int^*(F, E)$, then $(U, E)\tilde{\subset}\ \tilde{s}Cl(U, E)\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(F, E))\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(U, E))$ and so $(U, E) \in \tilde{S}S^*\tilde{I}O(X)_E$. Since $\tilde{s}Cl(U, E) = \tilde{s}Cl(\tilde{s}Int^*(F, E)) = (F, E)$, it follows that $\tilde{S}R\tilde{I}C(X)_E\tilde{\subset}\{\tilde{s}Cl(U, E) : (U, E) \in \tilde{S}S^*\tilde{I}O(X)_E\}$. Again, $(F, E) \in \tilde{S}S^*\tilde{I}O(X)_E$ implies that $(F, E) \subset \tilde{s}Cl(\tilde{s}Int^*(F, E))$. Now $\tilde{s}Cl(F, E)\tilde{\subset}\ \tilde{s}Cl(\tilde{s}Int^*(F, E))\tilde{\subset}\tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F, E)))\tilde{\subset}\tilde{s}Cl(F, E)$. So $\tilde{s}Cl(F, E) = \tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl(F, E)))$. Hence $\tilde{s}Cl(F, E)$ is soft $R - \tilde{I}-$closed. Therefore $\{\tilde{s}Cl(U, E) : (U, E) \in \tilde{S}S^*\tilde{I}O(X)_E\}\tilde{\subset}\tilde{S}R\tilde{I}C(X)_E$. □

3 Soft extremally disconnected spaces

In this section we give some properties of soft extremally disconnected spaces.

Theorem 3.1. *Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space with a soft co-dense ideal. Then the following are equivalent.*

(i) $(X, \tilde{\tau}, E)$ *is soft extremally disconnected;*

(ii) $(X, \tilde{\tau}^*, E)$ *is soft extremally disconnected.*

Proof. (i) $\Longrightarrow$(ii). Suppose $(X, \tilde{\tau}, E)$ is soft extremally disconnected and (U, E) is a soft regular open set in $(X, \tilde{\tau}^*, E)$. Then $(U, E) = \tilde{s}Int^*(\tilde{s}Cl^*(U, E))$. Since $\tilde{I}$ is soft co-dense, then $\tilde{s}Cl^*(U, E) = (U, E)^* = \tilde{s}Cl(U, E)$, by Corollary 2.1. So $(U, E) = \tilde{s}Int^*(\tilde{s}Cl^*(U, E)) = \tilde{s}Int^*(\tilde{s}Cl(U, E)) = \tilde{s}Int(\tilde{s}Cl(U, E))$ and thus, (U, E) is soft regular open in $(X, \tilde{\tau}, E)$. By hypothesis, $\tilde{s}Int(\tilde{s}Cl(U, E)) = \tilde{s}Cl(U, E)$ and therefore, $(U, E) = \tilde{s}Cl(U, E) = \tilde{s}Cl^*(U, E)$ and so (U, E) is soft $\tilde{\tau}^*$-closed. Hence by Lemma 1.4, $(X, \tilde{\tau}^*, E)$ is soft extremally disconnected.

(ii)$\Longrightarrow$(i). Let $(G, E) \in \tilde{\tau}$. Then $(G, E) \in \tilde{\tau}^*$. Since $(X, \tilde{\tau}^*, E)$ is soft extremally disconnected, $\tilde{s}Int^*(\tilde{s}Cl^*(G, E)) = \tilde{s}Cl^*(G, E)$ which implies that $\tilde{s}Int^*(\tilde{s}Cl(G, E)) = \tilde{s}Cl(G, E)$. Therefore $\tilde{s}Int(\tilde{s}Cl(G, E)) = \tilde{s}Cl(G, E)$ and so, $\tilde{s}Cl(G, E)$ is soft open. This shows that $(X, \tilde{\tau}, E)$ is soft extremally disconnected. □

The following example shows that the condition soft co-dense on the soft ideal cannot be removed.

Example 3.1. *(i) Let* $(X, \tilde{\tau}, E, \tilde{I})$ *be a soft ideal topological space, where* $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, \{(e, \{h_1\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_1, h_4\})\}, \{(e, \{h_1, h_3, h_4\})\}$ *and* $\tilde{I} = \{\tilde{\phi}, \{(e, \{h_1\})\}\}$. *Therefore* $\tilde{\tau}^* = \{\tilde{X}, \tilde{\phi}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_1, h_4\})\}, \{(e, \{h_4\})\}, \{(e, \{h_1\})\}, \{(e, \{h_3, h_4\})\}, \{(e, \{h_1, h_3, h_4\})\}, \{(e, \{h_2, h_3, h_4\})\}\}$. *Clearly, we have* $(X, \tilde{\tau}, E)$ *is soft extremally disconnected. If* $(F, E) = \{(e, \{h_1, h_4\})\}$, *then* $\tilde{s}Cl^*(F, E) = \{(e, \{h_1, h_2, h_4\})\}$ *which is not* $\tilde{\tau}^*$*-soft open. Therefore,* $(X, \tilde{\tau}^*, E)$ *is not soft extremally disconnected.*

(ii) Consider the soft ideal topological space, where $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, \{(e, \{h_1\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_3\})\}\}$ *and* $\tilde{I} = \{\tilde{?}, \{(e, \{h_1\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_3\})\}\}$. *Then* $\tilde{\tau}^* = \{\tilde{X}, \tilde{\phi}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_2, h_4\})\}, \{(e, \{h_1\})\}, \{(e, \{h_2, h_3, h_4\})\}, \{(e, \{h_1, h_2, h_4\})\}\}$. *Clearly,* $(X, \tilde{\tau}^*, E)$ *is soft extremally disconnected. But* $(X, \tilde{\tau}, E)$ *is not soft extremally disconnected, since* $\tilde{s}Cl(\{(e, \{h_1\})\}) = \{(e, \{h_1, h_2, h_4\})\}$ *is not soft open.*

Remark 3.1. *The family of all soft $R-\tilde{I}$-open sets of $(X,\tilde{\tau},E,\tilde{I})$ is a soft base for a soft topology , weaker than $\tilde{\tau}$, denoted by $\tilde{\tau}_{\tilde{I}}$. The family of all soft regular open sets of $(X,\tilde{\tau},E)$ is a soft base for a soft topology weaker than $\tilde{\tau}$, called soft semiregularization of $\tilde{\tau}$ and is denoted by $\tilde{\tau}_s$. The family of all soft $R-\tilde{I}$-closed sets of $(X,\tilde{\tau},E,\tilde{I})$ is a soft base for a soft topology denoted by $\tilde{\tau}_{\tilde{S}R\tilde{I}C}$. We have $\tilde{\tau}_s\tilde{\subset}\tilde{\tau}_{\tilde{I}}\tilde{\subset}\tilde{\tau}$.*

Theorem 3.2. *Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space with a soft co-dense ideal. Then the following are equivalent.*

(i) $(X,\tilde{\tau},E)$ is soft extremally disconnected.

(ii) $\tilde{S}R\tilde{I}C(X)_E=\tilde{S}R\tilde{I}O(X)_E$ and so $\tilde{S}R\tilde{I}C(X)_E$ is a base for $\tilde{\tau}_{\tilde{I}}$.

(iii) $\tilde{\tau}_{\tilde{S}R\tilde{I}C}=\tilde{\tau}_{\tilde{I}}$.

(iv) $\tilde{S}R\tilde{I}C(X)_E\tilde{\subset}\tilde{\tau}_{\tilde{I}}$.

Proof. (i) $\Longrightarrow$(ii). If $(F,E)\in\tilde{S}R\tilde{I}C(X)_E$, then, by Theorem 3.1, $(F,E)=\tilde{s}Cl(\tilde{s}Int^*(F,E))=\tilde{s}cl^*(\tilde{s}Int^*(F,E))=\tilde{s}Int^*(\tilde{s}Cl^*(\tilde{s}Int^*(F,E))=\tilde{s}Int^*(\tilde{s}Cl(\tilde{s}Int^*(F,E)))=\tilde{s}Int^*(F,E)$ and so (F,E) is $\tilde{\tau}^*$-soft open. Since (F,E) is $\tilde{\tau}$-soft closed, then (F,E) is $\tilde{\tau}^*$-soft closed. Also, $\tilde{s}Int(\tilde{s}Cl(F,E))=\tilde{s}Int^*(\tilde{s}Cl^*(F,E))=\tilde{s}Int^*(F,E)=(F,E)$, since (F,E) is both $\tilde{\tau}^*$-soft closed and $\tilde{\tau}^*$-soft open. Hence $(F,E)\in\tilde{S}R\tilde{I}O(X)_E$. Conversely, if $(F,E)\in\tilde{S}R\tilde{I}O(X)_E$, then $(F,E)=\tilde{s}Int(\tilde{s}Cl^*(F,E))$. Now, we have that $\tilde{s}Cl(\tilde{s}Int^*(F,E))=\tilde{s}Cl(\tilde{s}Int^*(\tilde{s}Cl^*(F,E)))=\tilde{s}Cl^*(\tilde{s}Int^*(\tilde{s}Cl^*(F,E)))=\tilde{s}Int^*(\tilde{s}Cl^*(F,E))=\tilde{s}Int(\tilde{s}Cl^*(F,E))=(F,E)$. Hence $(F,E)\in\tilde{S}R\tilde{I}C(X)_E$. Thus $\tilde{S}R\hat{I}O(X)_E=\hat{S}R\hat{I}C(X)_E$. Hence $\tilde{S}R\tilde{I}C(X)_E$ is a base for $\tilde{\tau}_{\tilde{I}}$.

(ii)$\Longrightarrow$(iii) and (iii) $\Longrightarrow$(iv) are obvious.

(iv)$\Longrightarrow$(i). Suppose (F,E) is a soft regular closed. Then (F,E) is soft closed and hence $(F,E)=\tilde{s}Cl(\tilde{s}Int^*(F,E))$, by Corollary 2.1, which implies that $(F,E)\in\tilde{S}R\tilde{I}C(X)_E$. Since $\tilde{\tau}_{\tilde{I}}\subset\tilde{\tau}$, by (iv), $(F,E)\in\tilde{\tau}$. Hence $(X,\tilde{\tau},E)$ is soft extremally disconnected, by Lemma 1.4. □

Theorem 3.3. *Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space. Then the following hold.*

(i) If $(X,\tilde{\tau},E)$ is soft extremally disconnected, then $\tilde{s}s\tilde{I}Cl(F,E)=\tilde{s}Cl(F,E)$ for every $(F,E)\in\tilde{\tau}_{\tilde{I}}$.

(ii) If $\tilde{S}R\tilde{I}C(X)_E\subset\tilde{\tau}$, then $(X,\tilde{\tau},E)$ is soft extremally disconnected.

Proof. (i) Suppose that $(X, \tilde{\tau}, E)$ is soft extremally disconnected and $(F, E) \in \tilde{\tau}_{\tilde{I}}$. Then $(F, E) \in \tilde{\tau}_{\tilde{I}} \subset \tilde{\tau} \subset \tilde{\tau}^*$. Since (F, E) is soft open, $\tilde{s}Cl(F, E)$ is soft open and hence $\tilde{\tau}^*$-soft open. Therefore $\tilde{s}s\tilde{I}Cl(F, E) = (F, E)\tilde{\cup}\tilde{s}Int^*(\tilde{s}Cl$ $(F, E) = \tilde{s}Cl(F, E)$.

(ii) Suppose $(F, E) \in \tilde{\tau}$. Since $\tilde{s}Cl(\tilde{s}Int^*(F, E)) \in \tilde{S}R\tilde{I}C(X)_E$, by hypothesis $\tilde{s}Int(\tilde{s}Cl(\tilde{s}Int^*(F, E))) = \tilde{s}Cl(\tilde{s}Int^*(F, E))$. Since $(F, E) \in \tilde{\tau}$, we have $\tilde{s}Int(\tilde{s}Cl(F, E)) = \tilde{s}Cl(F, E)$, which implies that $\tilde{s}Cl(F, E)$ is soft open. Hence $(X, \tilde{\tau}, E)$ is soft extremally disconnected. □

The converse of Theorem 3.3 need not be true as shown by the following example.

Example 3.2. *(i) Consider the soft ideal topological space of Example 2.6 (iii). Then we have $\tilde{\tau}^* = SS(X)_E$ and $\tilde{s}s\tilde{I}Cl(F, E) = \tilde{s}Cl(F, E)$ for every $(F, E) \in SS(X)_E$. If $(F, E) = \{(e, \{h_1\})\}$, then $\tilde{s}Cl(F, E) = \{(e, \{h_1, h_3, h_4$ $\})\}$ which is not soft open. Therefore, $(X, \tilde{\tau}, E)$ is not soft extremally disconnected.*

(ii) Consider the soft ideal topological space $(X, \tilde{\tau}, E, \tilde{I})$ where $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_3\})\}\}$ and $\tilde{I} = \{\tilde{\phi}, \{(e, \{h_3\})\}\}$. We can see that $(X, \tilde{\tau}, E)$ is soft extremally disconnected. If $(F, E) = \{(e, \{h_1, h_2, h_4\})\}$, then we obtain $\tilde{s}Cl(\tilde{s}Int^*(F, E)) = \tilde{s}Cl(F, E) = \{(e, \{h_1, h_2, h_4\})\}$ and so (F, E) is soft $R - \tilde{I}$-closed but not soft open.

Theorem 3.4. *Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space with a soft co-dense ideal $\tilde{I}$. Then the following hold.*

(i) If $\tilde{s}s\tilde{I}cl(F, E) = \tilde{s}Cl(F, E)$ for every $(F, E) \in \tilde{\tau}_{\tilde{I}}$. Then $(X, \tilde{\tau}, E)$ is soft extremally disconnected.

(ii) If $(X, \tilde{\tau}, E)$ is soft extremally disconnected, then $\tilde{S}R\tilde{I}C(X)_E \subset \tilde{\tau}$.

Proof. (i) Let (F, E) be a soft regular open set. Then $(F, E) = \tilde{s}Int(\tilde{s}Cl(F, E))$. Since $\tilde{I}$ is soft co-dense, by Corollary 2.1, $\tilde{s}Cl(F, E) = \tilde{s}s\tilde{I}Cl(F, E) = (F, E)\tilde{\cup}\tilde{s}Int^*(\tilde{s}Cl(F, E)) = \tilde{s}Int^*(F, E)\tilde{\cup}\tilde{s}Int^*(\tilde{s}Cl(F, E)) = \tilde{s}Int^*(\tilde{s}Cl(F, E))$ $= \tilde{s}Int(\tilde{s}Cl(F, E)) = (F, E)$. Therefore, $(X, \tilde{\tau}, E)$ is soft extremally disconnected by Lemma 1.3.

(ii) Suppose that $(F, E) \in \tilde{S}R\tilde{I}C(X)_E$. Then $(F, E) = \tilde{s}Cl(\tilde{s}Int^*(F, E))$. Since (F, E) is soft closed, by Corollary 2.1, $(F, E) = \tilde{s}Cl(\tilde{s}Int(F, E))$ and so (F, E) is soft regular closed. Therefore, $(F, E) \in \tilde{\tau}$, by Lemma 1.3. □

4 Soft $A^*_{\tilde{I}}$-continuous functions

In this section, we give two decompositions of soft continuity in terms of soft $\alpha-\tilde{I}$-continuity, soft pre $-\tilde{I}$-continuity and soft $A^*_{\tilde{I}}-$ continuity.

Definition 4.1. *Let $(X_1, \tilde{\tau}_1, E_1, \tilde{I})$ be a soft ideal topological space and $(X_2, \tilde{\tau}_2, E_2)$ be a soft topological space. The soft function $f_{up} : SS(X_1)_{E_1} \to SS(X_2)_{E_2}$ is called soft $A^*_{\tilde{I}}$-continuous if ${f_{up}}^{-1}(V, E) \in \tilde{S}A^*_{\tilde{I}}(X_1)_{E_1} \forall (F, E) \in \tilde{\tau}_2$.*

Example 4.1. *Let $(X_1, \tilde{\tau}_1, E_1, \tilde{I})$ be a soft ideal topological space and $(X_2, \tilde{\tau}_2, E_2)$ be a soft topological space where $X_1 = \{h_1, h_2\}$, $X_2 = \{x_1, x_2\}$, $E_1 = E_2 = \{e_1, e_2\}$, $\tilde{\tau}_1 = \{\tilde{X}_1, \tilde{\phi}, \{(e_1, \{h_2\})\}, \{(e_1, \tilde{X}_1), (e_2, \{h_2\})\}, \{(e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\}$, $\tilde{\tau}_2 = \{\tilde{X}_2, \tilde{\phi}, \{(e_1, \{x_1\}), (e_2, \{x_2\})\}, \{(e_1, \{x_1\}), (e_2, \tilde{X}_2)\}\}$ and $\tilde{I} = \{\tilde{\phi}, \{(e_1, \{h_2\})\}$. We get that $\tilde{S}A^*_{\tilde{I}}(\tilde{X}_1) = \{\tilde{X}_1, \tilde{\phi}, \{(e_1, \{h_2\})\}, \{(e_1, \tilde{X}_1), (e_2, \{h_2\})\}, \{(e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \tilde{X}_1)\}\}$. Let $u(h_1) = \{x_1\}, u(h_2) = \{x_2\}$ and $p(e_1) = e_1, p(e_2) = e_2$. Then the function f_{up} is soft $A^*_{\tilde{I}}$-continuous.*

Theorem 4.1. *The following properties are equivalent for a soft function $f_{up} : (X_1, \tilde{\tau}_1, E_1, \tilde{I}) \to (X_2, \tilde{\tau}_2, E_2)$:*

(i) f_{pu} is soft continuous.

(ii) f_{up} is both soft $\alpha-\tilde{I}$-continuous and soft $A^*_{\tilde{I}}$-continuous.

(iii) f_{up} is both soft pre $-\tilde{I}$-continuous and soft $A^*_{\tilde{I}}$- continuous.

Proof. Follows from Theorem 2.5. □

Conclusion 4.1. *Two types of soft sets in soft ideal topological spaces, called soft $R-\tilde{I}-$ closed and soft $A^*_{\tilde{I}}$-sets, are introduced and some of its properties are discussed. The concept of soft condense ideals is characterized by these collections of sets. It is shown, in Theorem 2.6, that a soft ideal $\tilde{I}$ is soft co-dense if and only if $\tilde{S}A^*_{\tilde{I}}(X)_E \cap \tilde{I} = \tilde{\phi}$. Also, in Theorem 2.7, $\tilde{I}$ is soft co-dense if and only if $\tilde{S}R\tilde{I}C(X, \tilde{\tau}, E) = \tilde{S}R_{\tilde{I}}C(X, \tilde{\tau}^*, E)$. Theorem 4.1 give two decompositions of soft continuity in terms of soft $\alpha-\tilde{I}$-continuity, soft pre $-\tilde{I}$-continuity and soft $A^*_{\tilde{I}}$-continuity and some equivalent conditions concerning this topic are established here.*

Acknowledgement

The authors are grateful to the referees for their valuable comments and suggestions.

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