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A short note on Hardy-Rogers contractive conditions

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Abstract

In this writing, we give some theorem in metric space under cyclic coupled Hardy-Rogers contractive conditions. Also, we offer a different perspective on this condition in metric space involving a graph.

1 Introduction and Preliminaries

Banach (1922) [1] established a noted fixed point theorem (viz., Banach Contraction Principle (or, in short, BCP) which is the most noteworthy results of nonlinear analysis and declared the main source of metric fixed point theory. Later, in 1973, Hardy and Rogers [2] investigated some fixed point theorems to improve and generalize the BCP. Since then, a great number of researchers have utilized various types of Hardy-Rogers contractive to attain fixed point results. Kirk et al. (2003) [3] established the concepts of cyclic maps in a complete metric space (or, in short, CMS), also introduced some fixed point theorems in Banach space.

Definition 1.1. *[3] Let $N \neq \emptyset \subseteq W$, $M \neq \emptyset \subseteq W$. A map $f : W \to W$ is a cyclic (w.r.t. N and M) if $N \supseteq f(M)$ and $M \supseteq f(N)$.*

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For $T : W \times W \to W$, the conception of a coupled fixed point $(a, b) \in W \times W$ such that $T(a, b) = a$, $T(b, a) = b$ was initially presented by Bhaskar and Lakshmikantham [4]. Furthermore, the existence and uniqueness of a coupled fixed point for such a map provides the mixed monotone property on a partially ordered metric space were attempted. Hereat, several of papers in this subject have been dedicated to extensions and generalizations; see, for instance, [[5]- [13]] and the references therein.

Lately, Choudhury and Maity [14] defined notion of cyclic coupled Kannan type contraction, also gave some fixed point theorem in CMS.

Definition 1.2. *[14] (i) Let $W \neq \emptyset$. An element $(a, a) \in W \times W$ is called to be strong coupled fixed point if $a = T(a, a)$.*

(ii) Let $N \neq \emptyset \subseteq W$ and $M \neq \emptyset \subseteq W$. We say any function $T : W \times W \to W$ such that $T(a, b) \in M$ if $a \in N$, $b \in M$ and $T(a, b) \in N$ if $a \in M$, $b \in N$ a cyclic mapping w.r.t. N, M.

Udo-utun [15] instantly extended the work of [14] for Ciric-Type mappings. Thereafter, best proximity consequences for generalized cyclic coupled maps were first deduced by Kadwin and Marudai [16]. Erelong, Ansari et al. [17] gave improvement and generalization results established in [16] under $C-$class functions.

Inspired and motivated these facts, we propose a new concept, cyclic coupled Hardy-Rogers contractive conditions. We search the existence of strong coupled fixed point for such a map that enable us to generalize many well-known deductions in the litterateur. More precisely, our main results improve and unify the several fixed point results in CMS. Besides, we consider an example to illustrate our result. After all, we offer a different perspective on this condition in metric space involving a graph.

2 Fixed Point Theorem

Definition 2.1. *(Cyclic coupled Hardy-Rogers contractive condition) Let $N \neq \emptyset \subseteq W$ and $M \neq \emptyset \subseteq W$. We call a map $T : W \times W \to W$ a cyclic coupled Hardy-Rogers contractive condition w.r.t. N and M if T is cyclic w.r.t. N and M satisfying, for a, $a' \in N$, b, $b' \in M$, the following* *condition*

$$\begin{aligned}
& d\left(T(a,b), T\left(b', a'\right)\right) + d\left(T(b,a), T\left(a', b'\right)\right) \\
\leq\ & \gamma_1 \left[d(T(a,b), a) + d(T(b,a), b)\right] \\
& + \gamma_2 \left[d\left(T\left(b', a'\right), b'\right) + d\left(T\left(a', b'\right), a'\right)\right] \\
& + \gamma_3 \left[d\left(T\left(b', a'\right), a\right) + d\left(T\left(a', b'\right), b\right)\right] \\
& + \gamma_4 \left[d\left(T(a,b), b'\right) + d\left(T(b,a), a'\right)\right] \\
& + \gamma_5 \left[d\left(a, b'\right) + d\left(b, a'\right)\right] \qquad (2.1)
\end{aligned}$$

owns, here $i = \overline{1,5};\ \gamma_i \in R^+$ *such that* $\sum_{i=1}^{5} \gamma < 1$.

Theorem 2.1. *Let* $N \neq \emptyset$, $M \neq \emptyset$ *be two closed subsets of CMS* (W, d). *Let* $T : W \times W \to W$ *a cyclic coupled Hardy-Rogers contractive condition w.r.t.* N *and* M *and* $N \cap M \neq \emptyset$. *Then* T *holds a strong coupled fixed point in* $N \cap M$.

Proof. Let $a_0 \in N$, $b_0 \in M$. We construct sequences $\{a_n\}$, $\{b_n\}$ as follows

$$T(b_n, a_n) = a_{n+1},\ T(a_n, b_n) = b_{n+1} \ \text{ for } \forall n \geq 0. \qquad (2.2)$$

Let $\vartheta_n = d(a_{n-1}, b_n) + d(b_{n-1}, a_n)$. We have to prove that $\vartheta_n \to 0$ as $n \to \infty$.

By (2.1), (2.2), we hold

$$\begin{aligned}
& d(a_n, b_{n+1}) + d(b_n, a_{n+1}) \\
\leq\ & \gamma_1 \left[d(T(b_{n-1}, a_{n-1}), b_{n-1}) + d(T(a_{n-1}, b_{n-1}), a_{n-1})\right] \\
& + \gamma_2 \left[d(T(a_n, b_n), a_n) + d(T(b_n, a_n), b_n)\right] \\
& + \gamma_3 \left[d(T(a_n, b_n), b_{n-1}) + d(T(b_n, a_n), a_{n-1})\right] \\
& + \gamma_4 \left[d(T(b_{n-1}, a_{n-1}), a_n) + d(T(a_{n-1}, b_{n-1}), b_n)\right] \\
& + \gamma_5 \left[d(b_{n-1}, a_n) + d(a_{n-1}, b_n)\right] \\
\leq\ & \gamma_1 \left[d(a_n, b_{n-1}) + d(b_n, a_{n-1})\right] \\
& + \gamma_2 \left[d(b_{n+1}, a_n) + d(a_{n+1}, b_n)\right] \\
& + \gamma_3 \left[d(b_{n+1}, a_n) + d(a_n, b_{n-1}) + d(a_{n+1}, b_n) + d(b_n, a_{n-1})\right] \\
& + \gamma_5 \left[d(b_{n-1}, a_n) + d(a_{n-1}, b_n)\right], \qquad (2.3)
\end{aligned}$$

or,

$$\vartheta_{n+1} \leq \gamma_1 \vartheta_n + \gamma_2 \vartheta_{n+1} + \gamma_3 (\vartheta_n + \vartheta_{n+1}) + \gamma_5 \vartheta_n, \tag{2.4}$$

equivalently,

$$\vartheta_{n+1} \leq \xi \vartheta_n, \tag{2.5}$$

where, $\xi = \{\gamma_1 + \gamma_3 + \gamma_5\} / 1 - \{\gamma_2 + \gamma_3\}$. Repeating the aforementioned procedure, we enjoy

$$\vartheta_{n+1} \leq \xi \vartheta_n \leq \xi^2 \vartheta_{n-1} \leq \cdots \leq \xi^{n+1} \vartheta_0, \tag{2.6}$$

where, by hypothesis about coefficients γ_i, $i = \overline{1,5}$, $0 \leq \xi < 1$; herefrom,

$$\vartheta_n \to 0 \text{ as } n \to \infty. \tag{2.7}$$

By (2.1) and (2.2), we get

$$\begin{aligned}
& d(a_n, b_n) + d(b_n, a_n) \\
\leq\ & \gamma_1 [d(T(b_{n-1}, a_{n-1}), b_{n-1}) + d(T(a_{n-1}, b_{n-1}), a_{n-1})] \\
& + \gamma_2 [d(T(a_{n-1}, b_{n-1}), a_{n-1}) + d(T(b_{n-1}, a_{n-1}), b_{n-1})] \\
& + \gamma_3 [d(T(a_{n-1}, b_{n-1}), b_{n-1}) + d(T(b_{n-1}, a_{n-1}), a_{n-1})] \\
& + \gamma_4 [d(T(b_{n-1}, a_{n-1}), a_{n-1}) + d(T(a_{n-1}, b_{n-1}), b_{n-1})] \\
& + \gamma_5 [d(b_{n-1}, a_{n-1}) + d(a_{n-1}, b_{n-1})] \\
\leq\ & \gamma_1 [d(a_n, b_{n-1}) + d(b_n, a_{n-1})] + \gamma_2 [d(b_n, a_{n-1}) + d(a_n, b_{n-1})] \\
& + \gamma_3 [d(b_n, a_n) + d(a_n, b_{n-1}) + d(a_n, b_n) + d(b_n, a_{n-1})] \\
& + \gamma_4 [d(a_n, b_n) + d(b_n, a_{n-1}) + d(b_n, a_n) + d(a_n, b_{n-1})] \\
& + \gamma_5 \begin{bmatrix} d(b_{n-1}, a_n) + d(a_n, b_n) + d(b_n, a_{n-1}) \\ + d(a_{n-1}, b_n) + d(b_n, a_n) + d(a_n, b_{n-1}) \end{bmatrix},
\end{aligned} \tag{2.8}$$

or,

$$\kappa \vartheta_n \geq d(a_n, b_n) + d(b_n, a_n), \tag{2.9}$$

where, $\kappa = \{\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + 2\gamma_5\} / \{1 - \gamma_3 - \gamma_4 - \gamma_5\} < 1$.

Next, from (2.7) and (2.9), we have

$$\begin{aligned}
& d(a_n, a_{n+1}) + d(b_n, b_{n+1}) \\
\leq\ & d(a_n, b_n) + d(b_n, a_n) + d(b_n, a_{n+1}) + d(a_n, b_{n+1}) \\
\to\ & 0 \text{ as } n \to \infty.
\end{aligned} \tag{2.10}$$

This testifies that $\{a_n\}$, $\{b_n\}$ are Cauchy sequences, thus are convergent.

As N and M be two closed subsets, $\{a_n\} \subseteq N$, and $\{b_n\} \subseteq M$, we have

$$a_n \to a \in N,\ b_n \to b \in M \text{ as } n \to \infty. \tag{2.11}$$

By (2.9), $d(a_n, b_n) \to 0$ as $n \to \infty$. Thusly, from (2.11),

$$a = b. \tag{2.12}$$

Herefrom $a \in N \cap M \neq \emptyset$.

Now, by (2.1), (2.2), we obtain

$$\begin{aligned}
& d(a, T(a,b)) + d(b, T(b,a)) \\
\leq\ & d(a, a_{n+1}) + d(a_{n+1}, T(a,b)) + d(b, b_{n+1}) + d(b_{n+1}, T(b,a)) \\
\leq\ & d(T(b_n, a_n), T(a,b)) + d(T(a_n, b_n), T(b,a)) \\
& + d(a, a_{n+1}) + d(b, b_{n+1}) \\
\leq\ & \gamma_1 [d(T(b_n, a_n), b_n) + d(T(a_n, b_n), a_n)] \\
& + \gamma_2 [d(T(a,b), a) + d(T(b,a), b)] \\
& + \gamma_3 [d(T(a,b), b_n) + d(T(b,a), a_n)] \\
& + \gamma_4 [d(T(b_n, a_n), a) + d(T(a_n, b_n), b)] \\
& + \gamma_5 [d(b_n, a) + d(a_n, b)] \\
& + d(a, a_{n+1}) + d(b, b_{n+1}).
\end{aligned} \tag{2.13}$$

Letting $n \to \infty$, by (2.12), $T(a,a) = a$; videlicet, we own a strong coupled fixed point of T. □

Corollary 2.1. *Let $N \neq \emptyset$, $M \neq \emptyset$ be two closed subsets of CMS (W, d). Let $T : W \times W \to W$ a cyclic coupled contractive condition w.r.t. N and M if T is cyclic w.r.t. N and M satisfying, for a, $a^{'} \in N$, b, $b^{'} \in M$, the following condition*

$$d\left(T(a,b), T\left(b^{'}, a^{'}\right)\right) + d\left(T(b,a), T\left(a^{'}, b^{'}\right)\right) \leq \gamma_5 \left(d\left(a, b^{'}\right) + d\left(b, a^{'}\right)\right)$$

holds, where $\gamma_5 \in R^{+}$such that $\gamma_5 < 1$ and $N \cap M \neq \emptyset$. Then T holds a strong coupled fixed point in $N \cap M$.

Proof. To verify the above corollary it suffices to get $\gamma_i = 0$, $i = \overline{1,4}$ in Theorem 2.1. □

Remark 2.1. *(i) In Berinde's coupled fixed point consequences [18] for mixed monotone mapping on partially ordered metric spaces, it is assumed that T is a cyclic coupled contractive condition w.r.t. N and M and $N \cap M \neq \emptyset$.*

(ii) The previous corollary further can be used to generalize, enhance and flourish the results of [4], [19] and [20] under the same conditions.

(iii) In Theorem 2.1, we can easily see that cyclic coupled Chatterjea type contraction map and cyclic coupled Kannan type contraction map [14] is improved and extended.

Next, we furnish an illustrative example of cyclic coupled Hardy-Rogers contractive condition defined on CMS. The following instance is inspired by [[14], Example 6].

Example 2.1. *Let $W = \mathbb{R}$ and the metric $d : W \times W \to \mathbb{R}$ described by $\forall a$, $b \in W$; $d(a,b) = |a - b|$. Let $M = [0, \pi]$ and $N = [-\pi, 0]$. $N \neq \emptyset$, $M \neq \emptyset$ thereupon are closed subsets of W and $d(N, M) = 0$. Define $T : W \times W \to W$ by*

$$T(a,b) = \begin{cases} -\frac{1}{5}\left|b \sin \frac{1}{b}\right|, & \text{if } (a,b) \in M \times N, \\ 0, & \text{if } (a,b) \in N \times M, \\ 3, & \text{otherwise.} \end{cases}$$

Let $\gamma_i = i/1000$, $i = \overline{1,5}$.

It is facilely to observe that overall circumstances of Theorem 2.1 are verified. Using Theorem 2.1, we may infer that T own a strong coupled fixed point on W. In the circumstances, T holds a fixed point as $(0,0)$.

3 A different point of view on coupled Hardy-Rogers contractive condition

Next, we present the existence for coupled fixed point of Hardy-Rogers condition by using Bhaskar&Lakshmikantham's definition in CMS via G.

Let Δ be a diagonal of $W \times W$, G be a graph with no parallel edges such that the set $V(G)$ of its vertices overlaps via W, $\Delta \subseteq E(G)$, here $E(G)$

is the set of the edges of G. Nominately, G is described by $(V(G), E(G))$. Farther, express by G^{-1} derived from G by overturning the direction of the edges in G. Consequently, $E\left(G^{-1}\right) = \{(a,b) \in W \times W : (b,a) \in E(G)\}$.

Definition 3.1. *[21] $T : W \times W \to W$ is edge-preserving if* $\left(a, b^{'}\right) \in E(G)$, $\left(b, a^{'}\right) \in E\left(G^{-1}\right)$ *implies* $\left(T(a,b), T\left(b^{'}, a^{'}\right)\right) \in E(G)$, $\left(T(b,a), T\left(a^{'}, b^{'}\right)\right) \in E\left(G^{-1}\right)$.

Theorem 3.1. *Let (W,d) be CMS via G, $F : W \times W \to W$ be edge-preserving and assume that the following features belong:*

1. *there exists $a_0, b_0 \in W$ such that $(a_0, T(a_0, b_0)) \in E(G)$ and $(b_0, T(b_0, a_0)) \in E\left(G^{-1}\right)$;*
2. *W holds the below feature:*

   (i) if any sequence $\{a_n\} \subseteq W$ such that for $\forall n \in N$; $a_n \to a$, $(a_n, a_{n+1}) \in E(G)$, then $\forall n \in N$; $(a_n, a) \in E(G)$,

   (ii) if any sequence $\{b_n\} \subseteq W$ such that $\forall n \in N$; $b_n \to b$, $(b_n, b_{n+1}) \in E\left(G^{-1}\right)$, then $\forall n \in N$; $(b_n, b) \in E\left(G^{-1}\right)$;
3. *there exists γ_i, $i = \overline{1,5}$ with $\sum_{i=1}^{5} \gamma_i < 1$ such that*

$$
\begin{aligned}
d\left(T(a,b), T\left(b^{'}, a^{'}\right)\right) &+ d\left(T(b,a), T\left(a^{'}, b^{'}\right)\right) \\
&\leq \gamma_1 \left[d\left(T(a,b), a\right) + d\left(T(b,a), b\right)\right] \\
&+\gamma_2 \left[d\left(T\left(b^{'}, a^{'}\right), b^{'}\right) + d\left(T\left(a^{'}, b^{'}\right), a^{'}\right)\right] \\
&+\gamma_3 \left[d\left(T\left(b^{'}, a^{'}\right), a\right) + d\left(T\left(a^{'}, b^{'}\right), b\right)\right] \\
&+\gamma_4 \left[d\left(T(a,b), b^{'}\right) + d\left(T(b,a), a^{'}\right)\right] \\
&+\gamma_5 \left[d\left(a, b^{'}\right) + d\left(b, a^{'}\right)\right]
\end{aligned}
$$

for all $a^{'}, b^{'}, a, b \in W$ with $\left(a, b^{'}\right) \in E(G)$, $\left(b, a^{'}\right) \in E\left(G^{-1}\right)$.

Then there exist $a, b \in W$ such that $a = T(a,b)$, $b = T(b,a)$.

Proof. Let $K = W \times W$. It is facilely to infer that the map $\upsilon : K \times K \to [0, \infty)$ identified by

$$
\upsilon\left((a,b), \left(b^{'}, a^{'}\right)\right) = d\left(a, b^{'}\right) + d\left(b, a^{'}\right), \quad \text{for all } (a,b), \left(b^{'}, a^{'}\right) \in K,
$$

is a complete metric on K. Next, determine the map $F : K \to K$ by

$$F(a,b) = (T(a,b), T(b,a)), \quad \text{for all } (a,b) \in K.$$

Let G_K be a directed graph given by $G_K = (V(G_K), E(G_K))$, hereby $V(G_K) = K$ and

$$E(G_K) = \left\{ \left((a,b), \left(b', a'\right) \right) : \left(a, b'\right) \in E(G) \text{ and } \left(b, a'\right) \in E\left(G^{-1}\right) \right\}.$$

Let $(a,b), \left(b', a'\right) \in K$ such that $\left((a,b), \left(b', a'\right)\right) \in E(G_K)$. Then, $\left(a, b'\right) \in E(G)$ and $\left(b, a'\right) \in E\left(G^{-1}\right)$. Because T is edge-preserving, we enjoy $\left(T(a,b), T\left(b', a'\right)\right) \in E(G)$, $\left(T(b,a), T\left(a', b'\right)\right) \in E\left(G^{-1}\right)$. Accordingly, $\left((T(a,b), T(b,a)), \left(T\left(b', a'\right), T\left(a', b'\right)\right)\right) \in E(G_K)$. Thusly $(F(a,b)$, $\left(F\left(b', a'\right)\right) \in E\, G_K)$. Due to (1), we hold $((a_0,b_0), (T(a_0,b_0), T(b_0,a_0))) \in E(G_K)$. Thereby $((a_0,b_0), F(a_0,b_0)) \in E(G_K)$. Now, by hypothesis (3), we get for all $\alpha = (a,b)$ and $\beta = \left(b', a'\right)$,$\upsilon(F(\alpha), F(\beta)) \leq \gamma_1 \upsilon(F(\alpha), \alpha) + \gamma_2 \upsilon(F(\beta), \beta) + \gamma_3 \upsilon(F(\beta), \alpha) + \gamma_4 \upsilon(F(\alpha), \beta) + \gamma_5 \upsilon(\alpha, \beta)$,for $\forall \left(a, b'\right) \in E(G)$, $\left(b, a'\right) \in E\left(G^{-1}\right)$. Ultimately, by any sequence (a_n, b_n) in (K, υ), here if $(a_n, b_n)_{n \in N} \to (a,b)$, $((a_n,b_n), (a_{n+1}, b_{n+1}))_{n \in N} \in E(G_K)$, then $a_n \to a$, $b_n \to b$, $(a_n, a_{n+1}) \in E(G)$, $(b_n, b_{n+1}) \in E\left(G^{-1}\right)$. From (2), $(a_n, a) \in E(G)$, $(b_n, b) \in E\left(G^{-1}\right)$, $((a_n,b_n), (a,b)) \in E(G_K)$ for $\forall n \in N$. Herewith, we can see that all states of Theorem 3.1. □

Corollary 3.1. *[[22], Theorem 4.3] Let (W,d) be CMS via G, and let $F : W \times W \to W$ be edge-preserving and assume that the following features belong:*

1. *there exists $a_0, b_0 \in W$ such that $(a_0, T(a_0,b_0)) \in E(G)$ and $(b_0, T(b_0,a_0)) \in E\left(G^{-1}\right)$;*
2. *W holds the below features:*

   (i) if any sequence $\{a_n\} \subseteq W$ such that for $\forall n \in N$; $a_n \to a$, $(a_n, a_{n+1}) \in E(G)$, then $\forall n \in N$; $(a_n, a) \in E(G)$,

   (ii) if any sequence $\{b_n\} \subseteq W$ such that for $\forall n \in N$; $b_n \to b$, $(b_n, b_{n+1}) \in E\left(G^{-1}\right)$, then $\forall n \in N$; $(b_n, b) \in E\left(G^{-1}\right)$,

3. *there exists* γ_3, γ_4 *and* γ_5 *with* $\gamma_3 + \gamma_4 + \gamma_5 < 1$ *such that*

$$\begin{aligned} & d\left(T(a,b), T\left(b^{'}, a^{'}\right)\right) + d\left(T(b,a), T\left(a^{'}, b^{'}\right)\right) \\ \leq\ & \gamma_3 \left[d\left(T\left(b^{'}, a^{'}\right), a\right) + d\left(T\left(a^{'}, b^{'}\right), b\right)\right] \\ & + \gamma_4 \left[d\left(T(a,b), b^{'}\right) + d\left(T(b,a), a^{'}\right)\right] \\ & + \gamma_5 \left[d\left(a, b^{'}\right) + d\left(b, a^{'}\right)\right] \end{aligned}$$

for $\forall a, b, a^{'}, b^{'} \in W$ *with* $\left(a, b^{'}\right) \in E(G)$, $\left(b, a^{'}\right) \in E\left(G^{-1}\right)$.

Then there exist $a, b \in W$ *such that* $T(a,b) = a$, $T(b,a) = b$.

Corollary 3.2. *Let* (W,d) *be CMS via* G, *and let* $F : W \times W \to W$ *be edge-preserving and assume that the below features belong:*

1. *there exists* $a_0, b_0 \in W$ *such that* $(a_0, T(a_0, b_0)) \in E(G)$ *and* $(b_0, T(b_0, a_0)) \in E\left(G^{-1}\right)$;

2. W *holds the below features:*

   (i) if any sequence $\{a_n\} \subseteq W$ *such that for* $\forall n \in N$; $a_n \to a$ *and* $(a_n, a_{n+1}) \in E(G)$, *then* $\forall n \in N$; $(a_n, a) \in E(G)$,

   (ii) if any sequence $\{b_n\} \subseteq W$ *such that for* $\forall n \in N$; $b_n \to b$ *and* $(b_n, b_{n+1}) \in E\left(G^{-1}\right)$, *then* $\forall n \in N$; $(b_n, b) \in E\left(G^{-1}\right)$,

3. *there exists* $0 \leq \gamma_5 < 1$ *such that*

$$d\left(T(a,b), T\left(b^{'}, a^{'}\right)\right) + d\left(T(b,a), T\left(a^{'}, b^{'}\right)\right) \leq \gamma_5 \left[d\left(ra, b^{'}\right) + d\left(b, a^{'}\right)\right]$$

for all $a, b, a^{'}, b^{'} \in W$ *with* $\left(a, b^{'}\right) \in E(G)$, $\left(b, a^{'}\right) \in E\left(G^{-1}\right)$.

Then there exist $a, b \in W$ *such that* $T(a,b) = a$, $T(b,a) = b$.

Conclusion 3.1. *In this study advance, enrich and generalize some coupled fixed point consequences presented by Berinde (2011), Alaeidizaji and Parvaneh (2012), Radenović (2013), Choudhury and Maity (2014) and Klanarong and Suantai (2015). Withinside the future extent of the opinion,* *reader can demostrate some existence and uniqueness consequences for coupled coincidence point and common fixed point of the condition (2.1) endowed with a graph.*

Competing interests

The author declares that she has no competing interests.

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