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# Pair of derivations on prime ideals in rings with involution

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**Abstract**

Let $R$ be a ring with involution $*$. An additive mapping $d : R \to R$ is said to a derivation of $R$ if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. The aim of this paper is to study the $*$-identities involving a pair of derivations on prime ideals of rings with involution. Precisely, we prove that if a ring $R$ with involution $*$ admits two derivations $d_1$ and $d_2$ such that $[d_1(x), d_2(x^*)] - d_1(x) \circ x^* - x \circ d_2(x^*) \in P$ for all $x \in R$, where $P$ is a prime ideal of $R$ and $char(R/P) \neq 2$, then $d_1(R) \subseteq P$ or $d_2(R) \subseteq P$. Moreover, some related results are also discussed.

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# 1 Introduction

Throughout the paper, $R$ will denote an associative ring with center $Z(R)$. Recall that an ideal $P$ of $R$ is said to be prime if $P \neq R$ and for $x, y \in R$, $xRy \subseteq P$ implies that $x \in P$ or $y \in P$. Therefore, $R$ is called a prime ring if and only if $(0)$ is the prime ideal of $R$. For any $x, y \in R$, the symbol $[x, y]$ will denote the commutator $xy - yx$; while the symbol $x \circ y$ will stand for the anticommutator $xy + yx$. An additive mapping $x \mapsto x^*$ satisfying $(xy)^* = y^*x^*$ and $(x^*)^* = x$ is called an involution. A ring equipped with an involution is known as ring with involution or $*$-ring. An element $x$ in a ring with involution $*$ is said to be hermitian if $x^* = x$ and skew-hermitian if $x^* = -x$. The sets of all hermitian and skew-hermitian elements of $R$ will be denoted by $H(R)$ and $S(R)$, respectively. If $R$ is 2-torsion free then every $x \in R$ can be uniquely represented in the form $2x = h+k$ where $h \in H(R)$ and $k \in S(R)$. The involution is said to be of the first kind if $H(R) \subseteq Z(R)$, otherwise it is said to be of the second kind. We refer the reader to [2] for justification and amplification for the above mentioned notations and key definitions.

A map $d : R \to R$ is a derivation of a ring $R$ if $d$ is additive and satisfies $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. Recently, many authors have obtained commutativity of prime and semiprime rings admitting suitably constrained additive mappings, as automorphisms, derivations, skew derivations and generalized derivations acting on appropriate subsets of the rings. We first recall that for a nonempty subset $S$ of $R$, a mapping $f : S \to R$ is called centralizing if $[f(x), x] \in Z(R)$ for all $x \in S$, in the special case where $[f(x), x] = 0$ for all $x \in S$, the mapping $f$ is said to be commuting on $S$. In [9], Posner proved that if a prime ring $R$ admits a nonzero derivation $d$ such that $[d(x), x] \in Z(R)$ for all $x \in R$, then $R$ is commutative. Over the last few decades, several authors have subsequently refined and extended this classical theorem in various directions (see [1], [4], [5] and [8] where further references can be found).

The objective of the present paper is to study the various $*$-differential identities involving pair of derivations on prime ideals of rings with involution.

# 2 The Results

We shall do a great deal of calculation with commutators and anti-commutators, routinely using the following basic identities: For all $s, t, w \in R$;

$$[st, w] = s[t, w] + [s, w]t \text{ and } [s, tw] = t[s, w] + [s, t]w$$

$$s \circ (tw) = (s \circ t)w - t[s, w] = t(s \circ w) + [s, t]w$$
$$(st) \circ w = s(t \circ w) - [s, w]t = (s \circ w)t + s[t, w].$$

We start our investigation with the following lemmas which will be used frequently throughout the discussions.

**Lemma 2.1.** *[3, Lemma 2.3] Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $[x, x^*] \in P$ for all $x \in R$, then $R/P$ is a commutative integral domain.*

**Lemma 2.2.** *[6, Lemma 1] Let $R$ be a ring, $P$ be a prime ideal of $R$. If $d$ is a derivation of $R$ satisfies the condition $[d(x), x] \in P$ for all $x \in R$, then $d(R) \subseteq P$ or $R/P$ is a commutative integral domain.*

The first main result of this paper is the following:

**Theorem 2.1.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $d_1$ and $d_2$ are derivations of $R$ satisfying the condition $[d_1(x), d_2(x^*)] - d_1(x) \circ x^* - x \circ d_2(x^*) \in P$ for all $x \in R$, then $d_1(R) \subseteq P$ and $d_2(R) \subseteq P$.*

*Proof.* By the hypothesis, we have

$$[d_1(x), d_2(x^*)] - d_1(x) \circ x^* - x \circ d_2(x^*) \in P \tag{2.1}$$

for all $x \in R$. A direct linearization yields

$$[d_1(x), d_2(y^*)] + [d_1(y), d_2(x^*)] - d_1(x) \circ y^* - d_1(y) \circ x^* - x \circ d_2(y^*) - y \circ d_2(x^*) \in P \tag{2.2}$$

for all $x, y \in R$. Writing $xh$ instead of $x$ in (2.2), where $h \in H(R) \cap Z(R)$, we may obtain

$$d_1(h)[x, d_2(y^*)] + d_2(h)[d_1(y), x^*] - d_1(h)(x \circ y^*) - d_2(h)(y \circ x^*) \in P \tag{2.3}$$

for all $x, y \in R$. Replacing $x$ by $xk$ in (2.3), where $k \in S(R) \cap Z(R)$, and using the condition $S(R) \cap Z(R) \nsubseteq P$, we find that

$$d_1(h)[x, d_2(y^*)] - d_2(h)[d_1(y), x^*] - d_1(h)(x \circ y^*) + d_2(h)(y \circ x^*) \in P \tag{2.4}$$

for all $x, y \in R$. Combining (2.3) and (2.4), we get

$$2d_1(h)([x, d_2(y^*)] - x \circ y^*) \in P$$

for all $x, y \in R$. This further implies

$$d_1(h)([x, d_2(y)] - x \circ y) \in P$$

for all $x, y \in R$. Primeness of $P$ yields $d_1(h) \in P$ or $[x, d_2(y)] - x \circ y \in P$. Consider the case $[x, d_2(y)] - x \circ y \in P$ for all $x, y \in R$. In particular, for $x = k$, we have $2ky \in P$ for all $y \in R$. Using the conditions $char(R/P) \neq 2$ and $S(R) \cap Z(R) \nsubseteq P$, we can obtain $R \subseteq P$, which is not possible. Thus we are left with the only case $d_1(h) \in P$ for all $h \in H(R) \cap Z(R)$, which yields $d_1(k) \in P$ for all $k \in S(R) \cap Z(R)$. In the similar manner we can also find $d_2(k) \in P$ for all $k \in S(R) \cap Z(R)$. Now, substituting $xk$ in place of $x$ in (2.2) and using the fact that $d_1(k), d_2(k) \in P$, we get

$$k([d_1(x), d_2(y^*)] - [d_1(y), d_2(x^*)] - d_1(x) \circ y^* + d_1(y) \circ x^* - x \circ d_2(y^*) + y \circ d_2(x^*)) \in P$$

for all $x, y \in R$. Since $S(R) \cap Z(R) \nsubseteq P$, it follows that

$$[d_1(x), d_2(y^*)] - [d_1(y), d_2(x^*)] - d_1(x) \circ y^* + d_1(y) \circ x^* - x \circ d_2(y^*) + y \circ d_2(x^*) \in P \tag{2.5}$$

for all $x, y \in R$. Adding (2.2) and (2.5), we obtain

$$2([d_1(x), d_2(y^*)] - d_1(x) \circ y^* - x \circ d_2(y^*)) \in P$$

for all $x, y \in R$. This implies

$$[d_1(x), d_2(y)] - d_1(x) \circ y - x \circ d_2(y) \in P \tag{2.6}$$

for all $x, y \in R$. For $y = k$ (2.6) reduces to $2kd_1(x) \in P$ for all $x \in R$. This gives $d_1(R) \subseteq P$. Similarly, for $x = k$ (2.6) reduces to $2kd_2(y) \in P$ for all $y \in R$ and hence $d_2(R) \subseteq P$. Thereby the proof is completed. $\square$

**Corollary 2.1.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $d$ is a derivation of $R$ satisfying the condition $[d(x), d(x^*)] - d(x \circ x^*) \in P$ for all $x \in R$, then $d(R) \subseteq P$.*

**Corollary 2.2.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. Then there are no nonzero derivations $d_1$ and $d_2$ of $R$ satisfying the condition $[d_1(x), d_2(x^*)] - d_1(x) \circ x^* - x \circ d_2(x^*) = 0$ for all $x \in R$.*

**Corollary 2.3.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. Then there is no nonzero derivation $d$ of $R$ satisfying the condition $[d(x), d(x^*)] = d(x \circ x^*)$ for all $x \in R$.*

**Theorem 2.2.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $R$ admits derivations $d_1$ and $d_2$ such that $d_1(x) \circ x^* + x \circ d_2(x^*) - [x, x^*] \in P$ for all $x \in R$, then $R/P$ is a commutative integral domain.*

*Proof.* By the hypothesis, we have

$$d_1(x) \circ x^* + x \circ d_2(x^*) - [x, x^*] \in P \text{ for all } x \in R. \tag{2.7}$$

A linearizing (2.7) and using it, we get

$$d_1(x) \circ y^* + d_1(y) \circ x^* + x \circ d_2(y^*) + y \circ d_2(x^*) - [x, y^*] - [y, x^*] \in P \tag{2.8}$$

for all $x, y \in R$. Replacing $x$ by $xh$ in (2.8) and using it, we get

$$d_1(h)(x \circ y^*) + d_2(h)(y \circ x^*) \in P$$

for all $x, y \in R$, which gives

$$d_1(k)(x \circ y^*) + d_2(k)(y \circ x^*) \in P \tag{2.9}$$

for all $x, y \in R$. Replacing $x$ by $xk$ in (2.9), we have

$$d_1(k)(x \circ y^*) - d_2(k)(y \circ x^*) \in P \tag{2.10}$$

for all $x, y \in R$. Combining (2.9) and (2.10), we obtain

$$2d_1(k)(x \circ y^*) \in P \text{ for all } x, y \in R.$$

This implies

$$d_1(k)(x \circ y) \in P \text{ for all } x, y \in R.$$

Application of primeness of $P$ yields $d_1(k) \in P$ or $x \circ y \in P$. If $x \circ y \in P$ for all $x, y \in R$, then for $y = k$, we get $2xk \in P$ for all $x \in R$. Using the hypotheses of theorem, we arrive at $R \subseteq P$, which is not possible. Thus, we have $d_1(k) \in P$. Similarly, $d_2(k)$ also in $P$. Now replacing $x$ by $xk$ in (2.8) and using $d_1(k), d_2(k) \in P$ and $S(R) \cap Z(R) \nsubseteq P$, we can find

$$d_1(x) \circ y^* - d_1(y) \circ x^* + x \circ d_2(y^*) - y \circ d_2(x^*) - [x, y^*] + [y, x^*] \in P. \quad (2.11)$$

From (2.8) and (2.11), we have

$$2d_1(x) \circ y^* + 2x \circ d_2(y^*) - 2[x, y^*] \in P \text{ for all } x, y \in R.$$

This implies that

$$d_1(x) \circ y + x \circ d_2(y) - [x, y] \in P \text{ for all } x, y \in R. \quad (2.12)$$

Putting $y = k$ in (2.12) and using $d_1(k) \in P$, we get

$$2kd_1(x) \in P \text{ for all } x \in R,$$

which gives us $d_1(R) \subseteq P$. Hence (2.12) reduces to

$$x \circ d_2(y) - [x, y] \in P \text{ for all } x, y \in R.$$

Again for $x = k$ in the above, we can get $d_2(R) \subseteq P$. Therefore, (2.7) becomes $[x, x^*] \in P$ for all $x \in R$. Application of Lemma 2.1 yields $R/P$ is a commutative integral domain. □

**Corollary 2.4.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $R$ admits a derivation $d$ such that $d(x \circ x^*) - [x, x^*] \in P$ for all $x \in R$, then $R/P$ is a commutative integral domain.*

**Corollary 2.5.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. If $d_1$ and $d_2$ are derivations of $R$ satisfying the condition $d_1(x) \circ x^* + x \circ d_2(x^*) - [x, x^*] = 0$ for all $x \in R$, then $R$ is a commutative integral domain.*

**Corollary 2.6.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. If $d$ is a derivation of $R$ satisfying the condition $d(x \circ x^*) - [x, x^*] = 0$ for all $x \in R$, then $R$ is a commutative integral domain.*

**Theorem 2.3.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $d_1$ and $d_2$ are derivations of $R$ satisfying $d_1(x)d_2(x^*) - [x, x^*] \in P$ for all $x \in R$, then $R/P$ is a commutative integral domain.*

*Proof.* We have

$$d_1(x)d_2(x^*) - [x, x^*] \in P \tag{2.13}$$

for all $x \in R$. If $d_1(R) \subseteq P$ or $d_2(R) \subseteq P$, then result follows from Lemma 2.1. Next, we may now assume that $d_1(R) \nsubseteq P$ and $d_2(R) \nsubseteq P$. Linearizing (2.13), we obtain

$$d_1(x)d_2(y^*) + d_1(y)d_2(x^*) - [x, y^*] - [y, x^*] \in P \tag{2.14}$$

for all $x, y \in R$. Replacing $x$ by $xh$ in (2.14), where $h \in H(R) \cap Z(R)$, we get

$$d_1(h)xd_2(y^*) + d_1(y)x^*d_2(h) \in P \tag{2.15}$$

for all $x, y \in R$. Replacing $x$ by $xk$ in (2.15), where $k \in S(R) \cap Z(R) \nsubseteq P$, then

$$d_1(h)xd_2(y^*) - d_1(y)x^*d_2(h) \in P \tag{2.16}$$

for all $x, y \in R$. Combining (2.15) and (2.16), we obtain

$$2d_1(h)xd_2(y^*) \in P$$

for all $x, y \in R$. This implies that

$$d_1(h)Rd_2(R) \subseteq P.$$

Since $d_2(R) \nsubseteq P$, primeness of $P$ yields $d_1(h) \in P$ for all $h \in H(R) \cap Z(R)$ and hence $d_1(k) \in P$ for all $k \in S(R) \cap Z(R)$. Similarly, we can find that $d_2(k) \in P$ for all $k \in S(R) \cap Z(R)$. Replacing $x$ by $xk$ in (2.14) and using $d_1(k), d_2(k) \in P$, one can get

$$d_1(x)d_2(y^*) - d_1(y)d_2(x^*) - [x, y^*] + [y, x^*] \in P \tag{2.17}$$

for all $x, y \in R$. Addition of (2.14) and (2.17) gives

$$2d_1(x)d_2(y^*) - 2[x, y^*] \in P.$$

That implies that

$$d_1(x)d_2(y) - [x, y] \in P \tag{2.18}$$

for all $x, y \in R$. Thus, in view of [7, Theorem 1(2)], $R/P$ is a commutative integral domain. □

**Corollary 2.7.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. If $d_1$ and $d_2$ are derivations of $R$ satisfying the condition $d_1(x)d_2(x^*) - [x, x^*] = 0$ for all $x \in R$, then $R$ is a commutative integral domain.*

**Theorem 2.4.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $R$ admits derivations $d_1$ and $d_2$ such that $[d_1(x) \circ x^*, d_2(x^*)] \in P$ for all $x \in R$, then one of the following holds:*

*(i)* $d_1(R) \subseteq P$

*(ii)* $d_2(R) \subseteq P$

*(iii)* $R/P$ *is a commutative integral domain.*

*Proof.* By the hypothesis, we have

$$[d_1(x) \circ x^*, d_2(x^*)] \in P \text{ for all } x \in R. \tag{2.19}$$

A linearization of (2.19) gives

$$\begin{aligned} &[d_1(x) \circ x^*, d_2(y^*)] + [d_1(x) \circ y^*, d_2(x^*)] + [d_1(x) \circ y^*, d_2(y^*)] \\ &+ [d_1(y) \circ x^*, d_2(x^*)] + [d_1(y) \circ x^*, d_2(y^*)] + [d_1(y) \circ y^*, d_2(x^*)] \in P \end{aligned} \tag{2.20}$$

for all $x, y \in R$. Replacing $x$ by $-x$ in (2.20), we get

$$\begin{aligned} &[d_1(x) \circ x^*, d_2(y^*)] + [d_1(x) \circ y^*, d_2(x^*)] - [d_1(x) \circ y^*, d_2(y^*)] \\ &+ [d_1(y) \circ x^*, d_2(x^*)] - [d_1(y) \circ x^*, d_2(y^*)] - [d_1(y) \circ y^*, d_2(x^*)] \in P \end{aligned} \tag{2.21}$$

for all $x, y \in R$. Combining (2.20) and (2.21) and using the hypothesis of theorem, we obtain

$$[d_1(x) \circ x^*, d_2(y^*)] + [d_1(x) \circ y^*, d_2(x^*)] + [d_1(y) \circ x^*, d_2(x^*)] \in P \tag{2.22}$$

for all $x, y \in R$. Replacing $y$ by $yh$ in (2.22), we find that

$$d_2(h)[d_1(x) \circ x^*, y^*] + d_1(h)[y \circ x^*, d_2(x^*)] \in P \text{ for all } x, y \in R.$$

In particular for $y = k$, we have

$$2kd_1(h)[x^*, d_2(x^*)] \in P \text{ for all } x \in R,$$

which gives that

$$d_1(h)[x, d_2(x)] \in P \text{ for all } x \in R.$$

Primeness of $P$ yields $d_1(h) \in P$ or $[x, d_2(x)] \in P$. In view of Lemma 2.2, the latter case gives that $d_2(R) \subseteq P$ or $R/P$ is a commutative integral domain. On the other hand, $d_1(h) \in P$ for all $h \in H(R) \cap Z(R)$ implies $d_1(k) \in P$ for all $k \in S(R) \cap Z(R)$. Now, taking $y = k$ in (2.22), we conclude that

$$-2k[d_1(x), d_2(x^*)] \in P \text{ for all } x \in R.$$

Using the hypotheses of theorem, we obtain

$$[d_1(x), d_2(x^*)] \in P \text{ for all } x \in R. \tag{2.23}$$

Linearization of (2.23) yields that

$$[d_1(x), d_2(y^*)] + [d_1(y), d_2(x^*)] \in P \text{ for all } x, y \in R. \tag{2.24}$$

Substituting $xh$ for $x$ in (2.24), where $0 \neq h \in S(R) \cap Z(R)$, we get

$$d_1(h)[x, d_2(y^*)] + d_2(h)[d_1(y), x^*] \in P \text{ for all } x, y \in R.$$

Taking $h = k^2$ in the last relation and using the given hypotheses of theorem, we obtain

$$d_1(k)[x, d_2(y^*)] + d_2(k)[d_1(y), x^*] \in P \text{ for all } x, y \in R.$$

Replacing $x$ by $xk$ in the last expression and using the condition $S(R) \cap Z(R) \not\subseteq P$, we get

$$d_1(k)[x, d_2(y^*)] - d_2(k)[d_1(y), x^*] \in P \text{ for all } x, y \in R. \tag{2.25}$$

Substituting $xk$ in place of $x$ in (2.24), where $0 \neq k \in S(R) \cap Z(R)$, we find that

$$d_1(k)[x, d_2(y^*)] + k[d_1(x), d_2(y^*)] - k[d_1(y), d_2(x^*)] - d_2(k)[d_1(y), x^*] \in P \tag{2.26}$$

for all $x, y \in R$. Using (2.25) and the condition $S(R) \cap Z(R)) \not\subseteq P$ in the above relation, we conclude that

$$[d_1(x), d_2(y^*)] - [d_1(y), d_2(x^*)] \in P \text{ for all } x, y \in R. \tag{2.27}$$

Addition of (2.24) and (2.27) yields that

$$2([d_1(x), d_2(y^*)]) \in P \text{ for all } x, y \in R.$$

That implies that

$$[d_1(x), d_2(y)] \in P \text{ for all } x, y \in R.$$

Hence, in view of [6, Theorem 1], we get the required results. Thereby the proof is completed now. □

**Corollary 2.8.** *Let $R$ be a ring with involution $*$ of the second kind, $P$ a prime ideal of $R$ such that $S(R) \cap Z(R) \nsubseteq P$ and $char(R/P) \neq 2$. If $R$ admits a derivation $d$ such that $[d(x) \circ x^*, d(x^*)] \in P$ for all $x \in R$, then $d(R) \subseteq P$ or $R/P$ is a commutative integral domain.*

**Corollary 2.9.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. If $R$ admits derivations $d_1$ and $d_2$ such that $[d_1(x) \circ x^*, d_2(x^*)] = 0$ for all $x \in R$, then one of the following holds:*

*(i)* $d_1 = 0$

*(ii)* $d_2 = 0$

*(iii)* *$R$ is a commutative integral domain.*

**Corollary 2.10.** *Let $R$ be a prime ring with involution $*$ of the second kind such that $char(R) \neq 2$. If $R$ admits a derivation $d$ such that $[d(x) \circ x^*, d(x^*)] = 0$ for all $x \in R$, then $R$ is a commutative integral domain or $d = 0$.*

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