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A new class of continuous functions via δgp-open sets in topological spaces

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Abstract

In this paper, a new class of almost continuity called almost δ gp-continuity is presented. Characterizations and properties of almost δ gp-continuous functions are discussed.

1 Introduction

The notion of continuity on topological spaces,as significant and fundamental subject in the study of topology, has been researched by many mathematicians. Several investigations related to almost continuity which is a generalization of continuity have been published.The study of almost continuity was initiated by Singal and Singal [29] in 1968. almost pre-continuous functions were introduced and investigated by Nasef and Noiri [22]. In this paper, we define and study the notion of almost δgp-continuous functions which is stronger than the notion of almost gpr-continuous functions [4]. Also,we obtain various characterizations of almost δgp-continuous functions and investigate some of their fundamental properties.

Throughout this paper, $(X,\tau),(Y,\sigma)$ and (Z,η) (or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed unless explicitly stated and $f:(X,\tau)\rightarrow(Y,\sigma)$ or simply $f:X\rightarrow Y$ denotes a function

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f of a topological space X into a topological space Y. Let $M \subseteq X$, then cl(M) = $\cap\{F : M \subseteq F$ and $F^c \in \tau\}$ is the closure of M. Also, $\int(M) = \cup\{O : O \subseteq M$ and $O \in \tau\}$ is the interior of M.

The class of δgp-open (resp. δgp-closed, open, closed, regular open, regular closed, δ-preopen, δ-semiopen, $e*$-open, preopen, semiopen and β-open) sets of (X,τ) containing a point p$\in$ X is denoted by δGPO(X,p)(resp. δGPC(X,p), O(X,p), C(X,p), RO(X,p), RC(X,p), δPO(X,p), δSO(X,p), $e*$O(X,p), PO(X,p), SO(X,p)and βO(X,p)).

2 Preliminaries

Definition 2.1. *A set M $\subseteq$ X is called pre-closed* [21] *(resp. regular-closed* [31]*, semi-closed* [19]*, β-closed* [1]*) if cl(int(M)) $\subseteq$ M (resp. M = cl(int(M)), int(cl(M)) $\subseteq$ M and int(cl(int(M)) $\subseteq$ M).*

Definition 2.2. *A set M $\subseteq$ X is called δ-closed* [35] *if M = $cl_\delta(M)$ where cl_δ(M) = $\{$ p $\in$ X :int(cl(N)) $\cap$ M $\neq \phi$, N $\in \tau$ and p $\in$ N $\}$.*

Definition 2.3. *A set M $\subseteq$ X is called δ-preclosed* [26] *(resp. e^*-closed* [13]*, δ-semiclosed* [25] *and a-closed* [14]*) if cl(int_δ(M)) $\subseteq$ M (resp. int(cl(int_δ(M)) $\subseteq$ M, int(cl_δ(M)) $\subseteq$ M and cl(int(cl_δ(M))) $\subseteq$ M).*

Definition 2.4. *A set M $\subseteq$ X is called:*
(i)δgp-closed [7]*(resp. gpr-closed* [17] *and gp-closed* [20]*) if pcl(M) $\subseteq$ G whenever M $\subseteq$ G and G is δ-open(resp. regular open and open) in X,*
(ii) gδs-closed [5] *if scl(M) $\subseteq$ G whenever M $\subseteq$ G and G is δ-open in X.*

The complements of the above mentioned closed sets are their respective open sets .

Definition 2.5. *A function $f : X \to$Y is called:*
(i) R-map [9]*(resp. δ-continuous* [23]*, almost continuous* [29]*, almost pre-continuous* [22]*, almost gp-continuous, almost gpr-continuous* [4] *and almost gδs-continuous* [6]*) if the inverse image of every regular open set G of Y is regular open (resp. δ-open, open, pre-open, gp-open, gpr-open and gδs-open) in X,*
(ii)δgp-continuous [32] *if the inverse image of every open set G of Y is δgp-open in X,*

(iii) almost contra continuous [3]*(resp. almost contra super-continuous* [11] *and contr R-map* [10]*) if the inverse image of every regular closed set G of Y is open(resp. δ-open and regular open) in X,*
(iv)almost perfectly-continuous [30]*) if the inverse image of every regular closed set G of Y is clopen in X,*
(v)almost contra δgp-continuous [34]*(resp. contra δgp-continuous* [33] *and δgp-irresolute* [32]*) if the inverse image of every regular open(resp. open and δgp-closed) set G of Y is δgp-closed in X.*

Definition 2.6. *A space X is said to be:*
(i)preregular$T_{\frac{1}{2}}$-space [16] *if GPRO(X)=PO(X),*
(ii) $T_{\delta gp}$-space [7] *if δGPO(X)=O(X),*
(iii)δgp$T_{\frac{1}{2}}$-space [7] *if δGPO(X)=PO(X),*
(iv) extremely disconnected [16] *if the closure of every open subset of X is open,*
(v) submaximal [27] *if every pre-open set is open,*
(vi)strongly irresolvable [15] *if every open subspace of X is irresolvable,*
(vii)nearly compact [28] *if every regular open cover of X has a finite subcover,*
(viii) r-T_1-space [12] *if for each pair of distinct points x and y of X, there exist regular open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$,*
(ix) r-T_2-space [12] *if for each pair of distinct points x and y of X, there exist regular open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$,*
(x) δgp-T_1-space [34] *if for any pair of distinct points p and q, there exist G,H $\in \delta$GPO(X) such that $p \in G$, $q \notin G$ and $q \in H$, $p \notin H$,*
(xi)δgp-Hausdorff space [33] *if for each pair of distinct points x and y of X, there exist G,H $\in \delta$GPO(X) such that $x \in G$, $y \in H$ and $G \cap H = \phi$,*
(xii) δgp-additive [33] *if δGPC(X) is closed under arbitrary intersections.*

Definition 2.7. [8] *A subset M of a space X is said to be N-closed relative to X if every cover of M by regular open sets of X has a finite subcover.*

Theorem 2.1. [33] *(i)If A and B are δgp-open subsets of a submaximal space X, then $A \cap B$ is δgp-open in X.*
(ii) Let X be a δgp-additive space.Then $A \subseteq X$ is δgp-closed if and only if δgp-cl(A)=A.

Definition 2.8. [18] *A space X is called locally indiscrete if O(X)=RO(X).*

Lemma 2.1. [24, 33] *Let (X,τ) be a space and let A be a subset of X. The following statements are true:*
(i) A $\in$ PO(X) if and only if scl(A) = int(cl(A)).
(ii) p $\in$ δgpcl(A) if and only if U $\cap$ A $\neq$ ϕ for every δgp-open set U containing p.

3 Almost δgp-Continuous Functions.

Definition 3.1. *A function f: X $\to$ Y is called almost δgp-continuous if f^{-1}(N) $\in$ δGPC(X) for each regular closed set N of Y.*

Theorem 3.1. *A function f: X $\to$ Y is almost δgp-continuous if and only if the inverse image of every regular open set of Y is δgp-open in X.*

Remark 3.1. *From Definitions 2.5 and 3.1,we have the following diagram for a function $f : X \to Y$:*

$$\begin{array}{ccccccccccccc} 1 & \longrightarrow & 2 & \longrightarrow & 3 & \longrightarrow & 4 & \longrightarrow & 5 & \longrightarrow & 6 & \longrightarrow & 7 \\ & & & & & & & & & & & & \uparrow \\ & & & & & & & & & & & & 8 \end{array}$$

Notations:
1- R-map; 2-δ-continuity; 3-almost continuity; 4-almost pre continuity;
5- almost gp-continuity; 6-almost δgp-continuity; 7-almost gpr-continuity;
8-δgp-continuous.

None of these implications is reversible.

Example 3.1. *Let X = {p,q,r,s}=Y, τ = {X, ϕ, {p}, {q}, {p,q}, {p,q,r}} and σ = {Y, ϕ, {p}, {q}, {p,q}, {p,r}, {p,q,r}}. Define f: (X,τ) $\to$ (Y,σ) by f(p) = f(r) = q , f(q) = p and f(s) = r. Clearly f is almost δgp-continuous but for {q}$\in$ RO(Y), f^{-1}({q}) = {p,r} $\notin$ GPO(X). Therefore f is not almost gp-continuous. Define g: (X,τ) $\to$ (Y,σ) by g(p) = p, g(q) = s, g(r) = r and g(s) = q.Then g is almost δgp-continuous but for {p} $\in$ O(Y), g^{-1}({p}) = {p} $\notin$ δGPO(X).Therefore g is not δgp-continuous. Define h: (X,τ) $\to$ (X,σ) by h(p) = h(q) = q, h(r) = p and h(s) = r.Then h is almost gpr-continuous but for {q} $\in$ RO(Y), h^{-1}({q}) = {p,q} $\notin$ δGPO(Y). Therefore h is not almost δgp-continuous*

Theorem 3.2. *If $f : X \to Y$ is almost δgp-continuous and Y is locally indiscrete space,then f is δgp-continuous.*

Proof. Let N be an open set in Y,then N is regular-open in Y. Since f is almost δgp-continuous, then f^{-1}(N) is δgp-open in X. Hence f is δgp-continuous □

Theorem 3.3. *Let X be a locally indiscrete space,then the following properties are equivalent:*

(i) *f:X→Y is almost gpr-continuous;*

(ii) *f:X→Y is almost δgp-continuous;*

(iii) *f:X→Y is almost gp-continuous.*

Proof. Follows from the Theorem 3.7 of [34] □

Theorem 3.4. *(i) If f:X→Y is almost gδs-continuous with X as extremely disconnected space,then it is almost δgp-continuous.*
(ii) If f:X→Y is almost δgp-continuous with X as strongly irresolvable space.Then it is almost gδs-continuous.

Proof. Follows from the Theorem 3.9 of [34]
As a consequence of Lemma 3.10 of [32],we have the following Theorem □

Theorem 3.5. *The following statements are equivalent:*

(i) *f: X → Y is almost perfectly continuous;*

(ii) *f: X → Y is almost contra continuous and almost pre-continuous;*

(iii) *f: X → Y is almost contra continuous and almost gp-continuous;*

(iv) *f: X → Y is almost contra super-continuous and almost δgp-continuous;*

(v) *f: X → Y is contra R-map and almost gpr-continuous;*

(vi) *f: X → Y is contra R-map and almost pre-continuous;*

(vii) *f: X → Y is almost contra super-continuous and almost pre-continuous.*

Theorem 3.6. *Let X be a $\delta gpT_{\frac{1}{2}}$-space. Then the following are equivalent:*

(i) *f: X → Y is almost pre-continuous;*

(ii) *f: X → Y is almost gp-continuous;*

(iii) *f: X $\to$ Y is almost δgp-continuous.*

Theorem 3.7. *Let X be a preregular$T_{\frac{1}{2}}$-space. Then the following statements are equivalent:*

(i) *f: X $\to$ Y is almost pre-continuous;*

(ii) *f: X $\to$ Y is almost gp-continuous;*

(iii) *f: X $\to$ Y is almost δgp-continuous;*

(iv) *f: X $\to$ Y is almost gpr-continuous.*

Theorem 3.8. *Let X be a $T_{\delta gp}$-space. Then the following are equivalent:*

(i) *f: X $\to$ Y is almost continuous;*

(ii) *f: X $\to$ Y is almost pre-continuous;*

(iii) *f: X $\to$ Y is almost gp-continuous;*

(iv) *f: X $\to$ Y is almost δgp-continuous;*

(v) *f: X $\to$ Y is almost gpr-continuous.*

Theorem 3.9. *The following are equivalent:*
(i) f: X $\to$ Y is almost δgp-continuous and X is δgp-additive;
(ii) for each p $\in$ X and each open set N containing f(p), there exists δgp-open set M containing p such that f(M) $\subset$ int(cl(N)).

Proof. Obvious □

Theorem 3.10. *The following statements are equivalent:*
(i) f: X $\to$ Y is almost δgp-continuous and X is δgp-additive;
(ii) For each p $\in$ X and each N$\in$O(Y,f(p)), there exists M $\in$ δGPO(X,p) such that f(M) $\subset$ scl(N);
(iii) For each p $\in$ X and each H$\in$RO(Y,f(p)),there exists G $\in$ δGPO(X,p) such that f(G) $\subset$ H;
(iv) For each p $\in$ X and each V$\in\delta$O(Y,f(p)),there exists U $\in$ δGPO(X,p) such that f(U) $\subset$ V;
(v) For each p $\in$ X and each V$\in\delta$C(Y,f(p)),there exists U $\in$ δGPC(X,p) such that f(U) $\subset$ V.

Proof. (i)$\longrightarrow$(ii): Let p $\in$ X and N be an open set of Y containing f(p). By (i) and Theorem 3.9, there exists M $\in$ δGPO(X,p) such that f(M) $\subset$ int(cl(N)). Since M is preopen,then by Lemma 2.1(i), f(M) $\subset$ scl(N).
(ii)$\longrightarrow$(iii): Let p $\in$ X and N$\in$RO(Y,f(p)). Then N $\in$O(Y,f(p)). By (ii), there exists M $\in$ δGPO(X,p) such that f(M) $\subset$ scl(N). Since H is preopen, then by Lemma 2.1(i), f(M) $\subset$ int(cl(N)) = N.
(iii)$\longrightarrow$(iv): Let p $\in$ X and N$\in\delta$O(Y,f(p)),then there exists M $\in$ O(X,f(p)) such that M $\subset$ int(cl(M)) $\subset$ N. Since int(cl(M))$\in$RO(Y,f(p)), by (iii), there exists U $\in$ δGPO(X,p) such that f(U) $\subset$ int(cl(M)) $\subset$ N.
(iv)$\longrightarrow$(i): Let p $\in$ X and N$\in$O(Y,f(p)). Then int(cl(N))$\in\delta$O(Y,f(p)).
By (iv), there exists M $\in$ δGPO(X,p) such that f(M) $\subset$ int(cl(N)).
Hence f is almost δgp-continuous.
(iv)$\leftrightarrow$(v): Obvious. □

Theorem 3.11. *Let X be a δgp-additive space.Then M $\subseteq$ X is δgp-closed(resp. δgp-open) if and only if δgp-cl(M)=M (resp. δgp-int(M)=M).*

Theorem 3.12. *The following statements are equivalent:*
(i) f: X $\to$ Y is almost δgp-continuous and X is δgp-additive;
(ii) f(δgp-cl(M)) $\subset$ cl_δ(f(M)) for each M $\subseteq$ X;
(iii) δgp-cl(f^{-1}(N)) $\subset$ f^{-1}(cl_δ(N)) for each N$\subseteq$Y;
(iv) f^{-1}(G)$\in\delta$GPC(X) for each G$\in\delta$C(Y);
(v) f^{-1}(H)$\in\delta$GPO(X) for each H$\in\delta$O(Y).

Proof. (i) $\longrightarrow$ (ii) Suppose that $N \in \delta C(Y)$ such that $f(M) \subset N$. Observe that $N = cl_\delta(N) = \bigcap\{F : N \subset F$ and $F \in RC(Y)\}$ and so $f^{-1}(N) = \bigcap\{f^{-1}(F) : N \subset F$ and $F \in RC(Y)\}$. By (i) and Definition 2.6(xii), we have $f^{-1}(N) \in \delta GPC(X)$ and $M \subset f^{-1}(N)$. Hence δgp-cl(M) $\subset$ f^{-1}(N), and it follows that f(δgp-cl(M)) $\subset$ N. Since this is true for any δ-closed set N containing f(M), we have f(δgp-cl(M)) $\subset$ cl$_\delta$(f(M)).
(ii)$\longrightarrow$(iii) Let D $\subset$ Y, then f^{-1}(D) $\subset$ X. By (ii),
f(δgp-cl(f^{-1}(D))) $\subset$ cl$_\delta$(f(f^{-1}(D))) $\subset$ δgp-cl(D). So that δgp-cl(f^{-1}(D)) $\subset$ f^{-1}(Cl$_\delta$(D)).
(iii)$\longrightarrow$(iv) Let G $\in$ δC(Y).Then by (iii), δgp-cl(f^{-1}(G)) $\subset$ f^{-1}(cl$_\delta$(G))=f^{-1}(G).
In consequence, δgp-cl(f^{-1}(G))=f^{-1}(G) and hence by Theorem 3.11, f^{-1}(G) $\in$ δGPC(X).
(iv)$\longrightarrow$(v):Clear.

(v)$\longrightarrow$(i): Let N $\in$ RO(Y).Then N $\in$ δO(Y).By (v),f^{-1}(N) $\in$ δGPO(X). Hence by Theorem 3.1, f is almost δgp-continuous □

Theorem 3.13. *The following statements are equivalent:*
(i) f: X $\rightarrow$ Y is almost δgp-continuous and X is δgp-additive;
(ii) For every N$\in$O(Y),f^{-1}(int(cl(N)$\in$$\delta$GPO(X);
(iii) For every M$\in$C(Y),f^{-1}(cl(int(M)$\in$$\delta$GPC(X);
(iv) For every N$\in$$\beta$O(Y),$\delta$gpcl($f^{-1}$(N)) $\subset$ f^{-1}(cl(N));
(v) For every M$\in$$\beta$C(Y),$f^{-1}$(int(M)) $\subset$ δgpint(f^{-1}(M));
(vi) For every M$\in$SC(Y),f^{-1}(int(M)) $\subset$ δgpint(f^{-1}(M));
(vii) For every N$\in$SO(Y),δgpcl(f^{-1}(N)) $\subset$ f^{-1}(cl(N));
(viii) For every M$\in$PO(Y),f^{-1}(M) $\subset$ δgpint(f^{-1}(int(cl(M)).

Proof. (i)$\longleftrightarrow$(ii): Let N $\in$ O(Y). Since int(cl(N)) $\in$ RO(Y) Then by (i), f^{-1}(int(cl(N)) $\in$ δGPO(X). The converse is similar.
(i)$\longleftrightarrow$(iii)It is similar to (i)$\longleftrightarrow$(ii).
(i)$\longrightarrow$ (iv): Let N $\in$ βO(Y),then cl(N) $\in$ RC(Y) so by(i),f^{-1}(cl(N)) $\in$$\delta$GPC(X). Since f^{-1}(N) $\subset$ f^{-1}(cl(N)) which implies δgpcl(f^{-1}(N)) $\subset$ f^{-1}(cl(N)).
(iv)$\longrightarrow$ (v) and (vi)$\longrightarrow$ (vii):Obvious
(v)$\longrightarrow$ (vi):It follows from the fact that SC(Y) $\subset$ βC(Y)
(vii)$\longrightarrow$ (i):It follows from the fact that RC(Y) $\subset$ SO(Y).
(i)$\longleftrightarrow$ (viii): Let N $\in$PO(Y). Since int(cl(N)) $\in$ RO(Y),then by (i), f^{-1}(int(cl(N))) $\in$ δGPO(X) and hence
f^{-1}(N) $\subset$ f^{-1}(int(cl(N))) = δgpint(f^{-1}(int(cl(N)))). Conversely,let N $\in$ RO(Y). Since N $\in$ PO(Y), f^{-1}(N) $\subset$ δgp-int(f^{-1}(int(Cl(N)))) =δgpint(f^{-1}(N)),in consequence, δgpint(f^{-1}(N))=f^{-1}(N) and by Theorem 3.11, f^{-1}(N) $\in$ δGPO(X). □

Theorem 3.14. *The following are equivalent:*
(i) f : X $\rightarrow$ Y is almost δgp-continuous and X is δgp-additive;
(ii) For every e^*-open set N of Y,f^{-1}(cl_δ(N)) is δgp-closed in X;
(iii) For every δ-semiopen subset N of Y,f^{-1}(cl_δ(N)) is δgp-closed set in X;
(iv) For every δ-preopen subset N of Y,f^{-1}(int(cl_δ(N))) is δgp-open set in X;
(v) For every open subset N of Y,f^{-1}(int(cl_δ(N))) is δgp-open set in X;
(vi) For every closed subset N of Y,f^{-1}(cl(int_δ(N))) is δgp-closed set in X.

Proof. (i)$\rightarrow$(ii):Let N $\in$ e^*O(Y) Then by Lemma 2.7 of [2], cl_δ(N) $\in$ RC(Y). By (i),f^{-1}(cl_δ(N)) $\in$ δGPC(X).

(ii)$\rightarrow$(iii):Obvious since δSO(Y) $\subset e^*$O(Y).
(iii)$\rightarrow$(iv):Let N $\in\delta$PO(Y),then int$_\delta$(Y\N) $\in \delta$SO(Y). By (iii),
f^{-1}(cl$_\delta$(int$_\delta$(Y\N)) $\in \delta$GPC(X) which implies f^{-1}(int(cl$_\delta$(N)) $\in \delta$GPO(X).
(iv)$\rightarrow$(v):Obvious since O(Y) $\subset \delta$PO(Y).
(v)$\rightarrow$(vi):Clear
(vi)$\rightarrow$(i):Let N$\in$RO(Y).Then N=int(cl$_\delta$(N)) and hence (Y\N) $\in$ C(X). By (vi),
f^{-1}(Y\N) = X\f^{-1}(int(cl$_\delta$(N))) = f^{-1}(cl(int$_\delta$(Y\N)) $\in \delta$GPC(X).
Thus f^{-1}(N) $\in \delta$GPO(X). □

Theorem 3.15. *The following are equivalent for a function f: X $\rightarrow$ Y:*
(i) f is almost δgp-continuous and X is δgp-additive;
(ii) For every e^*-open subset G of Y, f^{-1}(a-cl(G)) is δgp-closed set in X;
(iii) For every δ-semiopen subset G of Y, f^{-1}(δ-pcl(G)) is δgp-closed set in X;
(iv) For every δ-preopen subset G of Y, f^{-1}(δ-scl(G))) is δgp-open set in X.

Proof. Follows from the Lemma 3.1 of [2] □

Theorem 3.16. *If f:X $\rightarrow$ Y is an almost δgp-continuous injective function and Y is r-T_1, then X is δgp-T_1.*

Proof. Let (Y,σ) be r-T_1 and p, q $\in$ X with p $\neq$ q. Then there exist regular open subsets G, H in Y such that f(p) $\in$ G, f(q) $\notin$ G, f(p) $\notin$ H and f(q) $\in$ H. Since f is almost δgp-continuous, f^{-1}(G) and f^{-1}(H) $\in \delta$GPO(X) such that p $\in f^{-1}$(G), q $\notin f^{-1}$(G), p $\notin f^{-1}$(H) and q $\in f^{-1}$(H). Hence X is δgp-T_1 . □

Theorem 3.17. *If f:X $\rightarrow$ Y is an almost δgp-continuous injective function and Y is r-T_2, then X is δgp-T_2.*

Proof. Similar to the proof of Theorem 3.16 □

Theorem 3.18. *If f,g:X $\rightarrow$ Y are almost δgp-continuous with X as submaximal and δgp-additive and Y is Hausdorff, then the set $\{x \in X : f(x) = g(x)\}$ is δgp-closed in X.*

Proof. Let E = {x $\in$ X : f(x) = g(x)} and x $\notin$ (X\E). Then f(x)$\neq$ g(x). Since Y is Hausdorff, there exist open sets V and W of Y such that f(x) $\in$ V, g(x)$\in$ W and V $\cap$ W = ϕ, hence int(cl(V)) $\cap$ int(cl(W)) = ϕ. Since f and g are almost δgp-continuous, there exist G,H $\in \delta$GPO(X,x)) such that f(G) $\subseteq$ int(cl(V)) and g(H) $\subseteq$ int(cl(W)). Now, put U = G $\cap$ H, then U $\in \delta$GPO(X,x)) and f(U) $\cap$ g(U) $\subseteq$ int(cl(V)) $\cap$

int(cl(W)) =ϕ. Therefore, we obtain U $\cap$ E = ϕ and hence x $\notin$ δgpcl(E) then E = δgpcl(E). Since X is δgp-additive, E is δgp-closed in X. □

Definition 3.2. *A space X is called δgp-compact if every cover of X by δgp-open sets has a finite subcover.*

Definition 3.3. *A subset M of a space X is said to be δgp-compact relative to X if every cover of M by δgp-open sets of X has a finite subcover.*

Theorem 3.19. *If f:X $\rightarrow$ Y is almost δgp-continuous and K is δgp-compact relative to X, then f(K) is N-closed relative to Y.*

Proof. Let { A_α: $\alpha \in \Omega$ } be any cover of f(K) by regular open sets of Y . Then $\{f^{-1}(A_\alpha)$:$\alpha\in\Omega\}$ is a cover of K by δgp-open sets of X. Hence there exists a finite subset Ω_o of Ω such that K $\subset \cup\{f^{-1}(A_\alpha)$:$\alpha\in\Omega_o$ $\}$. Therefore, we obtain f(K) $\subset$ $\{A_\alpha$: $\alpha\in\Omega_o\}$. This shows that f(K) is N-closed relative to Y . □

Corollary 3.1. *If f:X $\rightarrow$ Y is an almost δgp-continuous surjection and X is δgp-compact and δgp-additive, then Y is nearly compact.*

Lemma 3.1. *Let X be δgp-compact. If A$\subset$X is δgp-closed, then A is δgp-compact relative to X.*

Proof. Let { B_α: $\alpha \in \Omega$ } be a cover of N by δgp-open sets of X. Note that (X-N) is δgp-open and that the set (X-N) $\cup$ { B_α: $\alpha \in \Omega$ } is a cover of X by δgp-open sets. Since X is δgp-compact, there exists a finite subset Ω_o of Ω such that the set (X-N) $\cup$ { B_α: $\alpha \in \Omega_o$ } is a cover of X by δgp-open sets in X.
Hence { B_α: $\alpha \in \Omega_o$ } is a finite cover of N by δgp-open sets in X. □

Theorem 3.20. *If the graph function $g : X \rightarrow X \times Y$ of $f : X \rightarrow Y$,defined by $g(x) = (x, f(x))$ for each $x \in X$ is almost δgp-continuous Then f is almost δgp-continuous.*

Proof. Let $N \in RO(Y)$, then $X \times V \in RO(X \times Y)$. As g is almost δgp-continuous, $f^{-1}(N) = g^{-1}(X \times N) \in \delta GPO(X)$. □

Theorem 3.21. *If the graph function $g : X \rightarrow X \times Y$ of $f : X \rightarrow Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$. If X is a submaximal space and δgp-additive, then g is almost δgp-continuous if and only if f is almost δgp-continuous.*

Proof. We only prove the sufficiency. Let x $\in$ X and W $\in$RO(X$\times$Y). Then there exist regular open sets U_1 and V in X and Y, respectively such that $U_1\times V \subset W$. If f is almost δgp-continuous, then there exists a δgp-open set U_2 in X such that x $\in$ U_2 and f(U_2)$\subset$V . Put U = ($U_2\cap U_2$).Then U is δgp-open and g(U) $\subset U_1\times V \subset W$. Thus g is almost δgp-continuous. □

Recall that for a function f:X $\to$ Y, the subset G_f=$\{$(x,f(x)):x $\in$X$\}$ $\subset$ X$\times$Y is said to be graph of f.

Definition 3.4. *A graph G_f of a function f:X $\to$ Y is said to be strongly δgp-closed if for each (p,q) $\notin G_f$, there exist U$\in\delta$GPO(X,p) and V$\in$RO(Y,q) such that (U$\times$V)$\cap$ G_f = ϕ.*

Lemma 3.2. *For a graph G_f of a function $f : X \to Y$, the following properties are equivalent:*
(i)G_f is strongly δgp-closed in $X \times Y$;
(ii)For each $(p, q) \notin G_f$, there exist $U \in \delta GPO(X, p)$ and $V \in RO(Y, q)$ such that f(U)$\cap V = \phi$.

Theorem 3.22. *Let f:X $\to$ Y have a strongly δgp-closed graph G_f. If f is injective, then X is δgp-T_1.*

Proof. Let x_1,$x_2\in$X with $x_1\neq x_2$.Then f(x_1)$\neq$f(x_2) as f is injective so that (x_1,f(x_2)) $\notin G_f$.Thus there exist U$\in\delta$GPO(X,x_1) and V$\in$RO(Y,f(x_2)) such that f(U)$\cap$V=ϕ. Then f(x_2)$\notin$f(U) implies $x_2\notin$U and it follows that X is δgp-T_1. □

Theorem 3.23. *(i)If f: X $\to$ Y is almost δgp-continuous and g: Y $\to$ Z is R-map, then (g$\circ$f):X $\to$ Z is almost δgp-continuous.*
(ii)If f: X $\to$ Y is δgp-continuous and g: Y $\to$ Z is almost continuous, then (g$\circ$f):X $\to$ Z is almost δgp-continuous.
(iii) If f: X $\to$ Y is δgp-irresolute and g: Y $\to$ Z is almost δgp-continuous,then (g$\circ$f):X $\to$ Z is almost δgp-continuous.

Proof. (i) Let N $\in$ RO(Z). Then g^{-1}(N) $\in$ RO(Y) since g is R-map. The almost δgp-continuity of f implies $f^{-1}[g^{-1}(\mathrm{N}))] = (g \circ f)^{-1}((\mathrm{N})) \in \delta$GPO(X).
Hence g$\circ$f is almost δgp-continuous.
The proofs of (ii) and (iii) are similar to (i). □

Definition 3.5. [33] *A function f: X $\to$ Y is called pre δgp-closed if f(U)$\in\delta$GPC(Y) for every U$\in\delta$GPC(X).*

Theorem 3.24. *If f: X $\rightarrow$ Y is a pre δgp-open surjection and g:Y $\rightarrow$ Z is a function such that g$\circ$f:X $\rightarrow$ Z is almost δgp-continuous, then g is almost δgp-continuous.*

Proof. Let y $\in$ Y and x $\in$ X such that f(x) = y. Let G $\in$ RO(Z,(g$\circ$f)(x)). Then there exists U $\in$ δGPO(X,x) such that such that g(f(U))$\subset$ G. Since f is pre δgp-open in Y, we have that g is almost δgp-continuous at y.

Let A be a subset of X. Then A is said to be H-closed [35] relative to X if for every cover { B_i: i $\in \Omega$ } of A by open sets of X, there exists a finite subset Ω_o of Ω such that A $\subset \cup\{$cl(B_i) : i $\in \Omega_o$ $\}$. □

Definition 3.6. *A function f: X $\rightarrow$ Y is said to be δgp^*- continuous if for each p $\in$ X and each N$\in$O(Y,f(p)), there exists M $\in$ δGPO(X,p) such that f(M) $\subset$ cl(N).*

Theorem 3.25. *If f: X $\rightarrow$ Y is δgp^*-continuous and K is δgp-compact relative to X, then f(K) is H-closed relative to Y.*

Proof. Similar to the proof of Theorem 3.19 □

Theorem 3.26. *If for each pair of distinct points p and q in a space X, there exists a function f of X into a Hausdorff space Y such that*
(i) f(p) $\neq$ f(q),
(ii) f is δgp^*-continuous at p and
(iii) almost δgp-continuous at q,then X is δgp-Hausdorff.

Proof. Since Y is Hausdorff, there exist open sets W_1 and W_2 of Y such that f(p)$\in$ W_1, f(q) $\in$ W_2 and $W_1 \cap W_2$ = ϕ, hence cl(W_1) $\cap$ int(cl(W_2)) = ϕ. Since f is δgp^*-continuous at p, there exists $U_1 \in$ δGPO(X,p) such that f(U_1) $\subset$ cl(W_1). Since f is almost δgp-continuous at q, there exists $U_2 \in \delta$GPO(X,q) such that f(U_2) $\subset$ int(cl(W_2)). Therefore, we obtain $U_1 \cap$ U_2 = ϕ. This shows that X is δgp-Hausdorff. □

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