The Aligarh Bulletin of Mathematics
Volume 39, Number 2 (2020), 17-26

ISSN: 0304-9787

On convergence of 2-dimensional fractional Elzaki transformation

S. D. Manjarekar
Department of Mathematics
L. V. H. Arts, Science and Commerce College
Nashik - 422003, India
Email: shrimathematics@gmail.com

(Received August 15, 2020)

Abstract

In this paper, we have extend the newly defined fractional Elzaki transformation to towards 2-Dimensional fractional Elzaki transformation. The fractional Elzaki transformation has simple relationship with fractional Laplace as well as Laplace transformation by the help of which one can solve fractional differential equations easily. We have found out the simple condition under which 2-dimensional fractional Elzaki transformation were convergent.

1 Introduction

The theory of modern integral transform [1, 3, 5, 9] which includes Fourier, Mellin, Laplace, Wavelet, Hilbert, Weirstrass, Chirplet, Abel's, Laplace – Steiltjes , Laplace — Carson, L_2– transform and ZZ transformation which plays an important role in the theory of fractional calculus.

The generalized definition of integral transform has been defined [7] in well manner under new conditions with its inversion and convolution property, relationship with other integral Transform which includes Elzaki – Tarig, Laplace, Mellin, L_2, Abel's, Weirstrass, Hilbert, Fourier, Laplace – Carson, Laplace – Stieltjes

Keywords and phrases : Integral Transformation, Fractional Transformations, Convergence of Integrals
2010 AMS Subject Classification : 44A10, 26A33, 40A10

Transformations has being established in well manner. All these Transformations have tremendous applications [8] in fractals, Bio- Mathematics, Computational Fluid Dynamics.

In this paper the fractional Elzaki [9] transformation has been extended to the two dimensional fractional Elzaki transformation and the Convergence has been discussed.Recently [8], we have define fractional Elzaki transformations along with its properties including Translation, Dilation, Convolution and applications towards the fractional differential Equations.

The paper mainly divided into two parts. In first part, some basic required definition were given. In second part the extended definition of fractional Elzaki transformation were given and its convergence were discussed and some concluding remarks were given at end of paper along with references.

2 Preliminaries

We give some definitions and their properties for our main results.

(1) **Laplace Type Integral Transform:**
Consider a function $f(x)$ which is piece wise continuous and of exponential order then the Laplace — type integral[1] transform is defined as follows:

$$L_\epsilon[f(x);p] = \int_0^\infty \epsilon^{'}(x)e^{\phi(p)\epsilon(x)}f(x)\,dx \tag{2.1}$$

Where $\phi(p)$ is invertible function with $\varepsilon\left(x\right) = \int e^{-a(x)}dx$ an exponential function and $a(x)$ as invertible function.

(2) **Generalized Elzaki — Tarig Transformation:**
We have consider the definition of Generalized Elzaki – Tarig Transformation [7] as;

$$\Im_\varepsilon\left\{f\left(x\right);p\right\} = \int\limits_0^\infty \Phi\left(\frac{1}{p}\right)\Phi_1\left(p\right)\varepsilon'\left(x\right)e^{-\Phi(p)\varepsilon(x)}f\left(x\right)dx \qquad ,p\neq 0 \tag{2.2}$$

Where $f(x) \in$ S and it is given by
S = $\left\{f\left(x\right) : \exists k_1, k_2 > 0, \left|f\left(x\right)\right| < Me^{\frac{|x|}{k_j}}, x \in \left(-1\right)^j X\left[0,\infty\right), M > 0\right\}$ and $\Phi\left(\frac{1}{p}\right), \Phi_1\left(p\right)$ are invertible functions of p with $\varepsilon\left(x\right) = \int e^{-a(x)}dx$

an exponential function and $a(x)$ as invertible function, thus from the definitions above it can be seen that it is the generalization of Elzaki – Tarig transformation.

(3) **Conformable fractional Exponential function:**
The conformable fractional exponential function [8] is defined as;

$$E_\alpha(c,t) = e^{c\frac{t^\alpha}{\alpha}} \tag{2.3}$$

with the condition $0 < \alpha \leq 1$ and $c \in \Re$.

(4) **1 -– D fractional Elzaki Transformation:**
Let $0 < \alpha \leq 1$ and $f : [0,\infty) \to \Re$ satisfying the condition the set in (2.2)then the conformable fractional Elzaki transform of order α of $f(x)$ is defined as [7];

$$E_\alpha{}^c[f(t)](p) = \int_0^\infty pK_\alpha(-p,t)f(t)d_\alpha t \tag{2.4}$$

where $K_\alpha(-p,t) = E_\alpha(-\frac{1}{p},t)$ and $d_\alpha t = t^{\alpha-1}dt$

3 Convergence of Two dimensional fractional Elzaki Transformation

The definition of 1 -– dimensional conformable fractional Elzaki transformation can be extend to 2– dimensional as follows;

2 — Dimensional conformable fractional Elzaki Transformation:
Given function, $f(x,t) : [0,\infty) \times [0,\infty) \to \Re$ be real valued function satisfying the condition in (2.2) independently then the 2 -– dimensional conformable fractional Elzaki transformation at a point (p,q) can be defined as;

$$E_\alpha{}^{2c}[f(t)](p,q) = pq\int_0^\infty\int_0^\infty e^{-(\frac{t^\alpha}{p\alpha}+\frac{t^\alpha}{q\alpha})}f(x,t)d_\alpha x d_\alpha t \tag{3.1}$$

Now we prove the convergence of this 2 -– dimensional fractional Elzaki transformation and find out the conditions under which it converges almost everywhere.

Theorem 3.1. *If f(x,t) is continuous on [0,∞) $\times$ [0,∞) and integral converges at $p = p_0$ and $q = q_0$ then the two-dimensional fractional Elzaki transform of f(x,t) converges for $\frac{1}{p} > \frac{1}{p_0}$ and $\frac{1}{q} > \frac{1}{q_0}$ where $\epsilon(p,q,x,t) = (\frac{t^\alpha}{p\alpha} + \frac{t^\alpha}{q\alpha}) > 0$ and increasing in the positive quadrant.*

Proof. : To prove the result we will prove following two lemmas and combined the result of them to get the desired result.

Lemma 3.1. *If* $\mathbb{T}_\epsilon\{f(x,t);q\} = \int_0^t qf(x,t)e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t$ *converges at* $q = q_0$ *then the integral converges for* $\frac{1}{q} > \frac{1}{q_0} > 0$.

Proof. Consider the following function:

$$\alpha(x,t) = \int_0^t q_0\phi(x,u)e^{-\frac{t^\alpha}{q_0{}^\alpha}}\mathrm{d}_\alpha u$$

where $0 < t < \infty$,then clearly by using above definition $\alpha(x,0) = 0$ and $\lim_{t\to\infty}\int_0^t q_0\phi(x,t)e^{-\frac{t^\alpha}{q_0{}^\alpha}}\mathrm{d}_\alpha u$ converges at $q = q_0$ and by fundamental theorem of calculus we get,

$$\alpha_t(x,t) = q_0 e^{-\frac{t^\alpha}{q_0{}^\alpha}}\phi(x,t)$$

which after rearranging it gives; $\phi(x,t) = \frac{1}{q_0}e^{\frac{t^\alpha}{q_0{}^\alpha}}\alpha_t(x,t)$.
Choose ϵ_1 and R_1 such that $0 < \epsilon_1 < R_1$, then the above integral becomes,

$$\begin{aligned}\int_{\epsilon_1}^{R_1} q\phi(x,t)e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t &= \int_{\epsilon_1}^{R_1}\frac{q}{q_0}\alpha_t(x,t)e^{\frac{t^\alpha}{q_0{}^\alpha}}e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t\\ &= \int_{\epsilon_1}^{R_1}\frac{q}{q_0}\alpha_t(x,t)e^{-t^\alpha(\frac{1}{q^\alpha}-\frac{1}{q_0{}^\alpha})}\mathrm{d}_\alpha t\end{aligned}$$

by applying integration by parts it comes out to be

$$\begin{aligned}&\int_{\epsilon_1}^{R_1} q\phi(x,t)e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t\\ &= \frac{q}{q_0}\left\{\left|e^{-t^\alpha(\frac{1}{q^\alpha}-\frac{1}{q_0{}^\alpha})}\alpha(x,t)\right|_{\epsilon_1}^{R_1} + \int_{\epsilon_1}^{R_1}(\frac{1}{q}-\frac{1}{q_0})\alpha(x,t)e^{-t^\alpha(\frac{1}{q^\alpha}-\frac{1}{q_0{}^\alpha})}\mathrm{d}t\right\}\\ &= \frac{q}{q_0}[e^{-R_1{}^\alpha(\frac{1}{q^\alpha}-\frac{1}{q_0{}^\alpha})}\alpha(x,R_1) - e^{-\epsilon_1{}^\alpha(\frac{1}{q^\alpha}-\frac{1}{q_0{}^\alpha})}\alpha(x,\epsilon_1)\end{aligned}$$

$$+ \int_{\epsilon_1}^{R_1} (\frac{1}{q} - \frac{1}{q_0}) \alpha(x,t) e^{-t^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} \mathrm{d}t]$$

by applying $\epsilon_1 \to 0$ and using $\alpha(x,0) = 0$, we get

$$\int_0^{R_1} q\phi(x,t) e^{-\frac{t^\alpha}{q^\alpha}} \mathrm{d}_\alpha t = \frac{q}{q_0} [e^{-R_1{}^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} \alpha(x, R_1) + \int_0^{R_1} (\frac{1}{q} - \frac{1}{q_0}) \alpha(x,t) e^{-t^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} \mathrm{d}t]$$

Taking ,$R_1 \to \infty$, if $\frac{1}{q} > \frac{1}{q_0}$, the first term inside the bracket on R.H.S. tends to zero, hence we get

$$\int_0^\infty q\phi(x,t) e^{-\frac{t^\alpha}{q^\alpha}} \mathrm{d}_\alpha t = (\frac{1}{q} - \frac{1}{q_0}) \int_0^\infty \alpha(x,t) e^{-t^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} \mathrm{d}t$$

which exits if $\frac{1}{q} > \frac{1}{q_0}$. □

Lemma 3.2. *If* $\mathbb{T}_\epsilon \{f(x,t); p\} = \int_0^x pf(x,t) e^{-\frac{x^\alpha}{p^\alpha}} \mathrm{d}_\alpha v$ *converges at* $p = p_0$ *then the integral converges for* $\frac{1}{p} > \frac{1}{p_0} > 0$.

Proof. Consider the following function:

$$\alpha(x,t) = \int_0^x p_0 \phi(v,t) e^{-\frac{x^\alpha}{p_0{}^\alpha}} \mathrm{d}_\alpha v$$

where $0 < x < \infty$,then clearly by using above definition $\alpha(0,t) = 0$ and $\lim_{x\to\infty} \int_0^x p_0 \phi(x,t) e^{-\frac{x^\alpha}{p_0{}^\alpha}} \mathrm{d}_\alpha v$ converges at $p = p_0$ and by fundamental theorem of calculus we get,

$$\alpha_x(x,t) = p_0 e^{-\frac{x^\alpha}{p_0{}^\alpha}} \phi(x,t)$$

which after rearranging it gives; $\phi(x,t) = \frac{1}{p_0} e^{\frac{x^\alpha}{p_0\alpha}} \alpha_x(x,t)$.
Choose ϵ_2 and R_2 such that $0 < \epsilon_2 < R_2$, then the above integral becomes,

$$\int_{\epsilon_2}^{R_2} p\phi(x,t)e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x = \int_{\epsilon_2}^{R_2} \frac{p}{p_0} \alpha_x(x,t) e^{\frac{x^\alpha}{p_0\alpha}} e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x$$
$$= \int_{\epsilon_2}^{R_2} \frac{p}{p_0} \alpha_x(x,t) e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \mathrm{d}_\alpha x$$

by applying integration by parts it comes out to be

$$\int_{\epsilon_2}^{R_2} p\phi(x,t)e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x$$
$$= \frac{p}{p_0} \left\{ \left| e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \alpha(x,t) \right|_{\epsilon_2}^{R_2} + \int_{\epsilon_2}^{R_2} (\frac{1}{p} - \frac{1}{p_0}) \alpha(x,t) e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \mathrm{d}x \right\}$$
$$= \frac{p}{p_0} [e^{-R_2{}^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \alpha(R_2,t) - e^{-\epsilon_2{}^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \alpha(\epsilon_2,t)$$
$$+ \int_{\epsilon_2}^{R_2} (\frac{1}{p} - \frac{1}{p_0}) \alpha(x,t) e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \mathrm{d}x]$$

by applying $\epsilon_2 \to 0$ and using $\alpha(0,t) = 0$, we get

$$\int_{0}^{R_2} p\phi(x,t)e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x$$
$$= \frac{p}{p_0} [e^{-R_2{}^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \alpha(R_2,t) + \int_{0}^{R_2} (\frac{1}{p} - \frac{1}{p_0}) \alpha(x,t) e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \mathrm{d}x]$$

Taking ,$R_2 \to \infty$, if $\frac{1}{p} > \frac{1}{p_0}$, the first term inside the bracket on R.H.S. tends to zero, hence we get

$$\int_{0}^{\infty} p\phi(x,t)e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x = (\frac{1}{p} - \frac{1}{p_0}) \int_{0}^{\infty} \alpha(x,t) e^{-x^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0\alpha})} \mathrm{d}$$

which exits if $\frac{1}{p} > \frac{1}{p_0}$. □

Thus by combining Lemma (3.2) and (3.3) the two-dimensional new integral transform of $f(x,t)$ converges on for $\frac{1}{p} > \frac{1}{p_0}$ and $\frac{1}{q} > \frac{1}{q_0}$ hence the theorem. □

4 Uniform Convergence of Two dimensional fractional Elzaki Transformation

Theorem 4.1. *If $f(x,t)$ is continuous on [0, ∞) $\times$ [0, ∞) and integral $\mathbb{H}(x,t) = p_0 q_0 \int\limits_0^x \int\limits_0^t f(u,v) e^{-\frac{x^\alpha}{p_0{}^\alpha}} e^{-\frac{t^\alpha}{q_0{}^\alpha}} \mathrm{d}u\mathrm{d}v$ is bounded on [0, ∞) $\times$ [0, ∞) then the two-dimensional new integral transform of $f(x,t)$ converges uniformly on [p,∞) $\times$ [q,∞) if $\frac{1}{p} > \frac{1}{p_0}$ and $\frac{1}{q} > \frac{1}{q_0}$.*

Proof. To prove the result we will prove following two lemmas and combined the result of them to get the desired result.

Lemma 4.1. *If $g(x,t) = q_0 \int\limits_0^t f(x,v) e^{-\frac{v^\alpha}{q_0{}^\alpha}} \mathrm{d}_\alpha v$ is bounded on [q_0, ∞) then the integral converges uniformly on [q , ∞) for $\frac{1}{q} > \frac{1}{q_0}$.*

Proof. Consider, $0 < R < R_1$ and the integral;

$$q \int\limits_R^{R_1} f(x,v) e^{-\frac{v^\alpha}{q^\alpha}} \mathrm{d}_\alpha v = q \int\limits_R^{R_1} e^{-\frac{v^\alpha}{q^\alpha}} e^{-\frac{v^\alpha}{q_0{}^\alpha}} e^{\frac{v^\alpha}{q_0{}^\alpha}} f(x,v) \mathrm{d}_\alpha v \tag{4.1}$$

After rearranging the terms it gives

$$q \int\limits_R^{R_1} f(x,v) e^{-\frac{v^\alpha}{q^\alpha}} \mathrm{d}_\alpha v = \frac{q}{q_0} \int\limits_R^{R_1} e^{-t^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} g_t(x,t) \mathrm{d}t$$

By using Fundamental Theorem of Calculus and applying integration by parts; it turns out to be

$$q \int\limits_R^{R_1} f(x,t) e^{-\frac{t^\alpha}{q^\alpha}} \mathrm{d}_\alpha t = \frac{q}{q_0} [g(x,R_1) e^{-R_1{}^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} - g(x,R) e^{-R^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})}$$
$$+ (\frac{1}{q} - \frac{1}{q_0}) \int\limits_R^{R_1} g(x,t) e^{-t^\alpha(\frac{1}{q^\alpha} - \frac{1}{q_0{}^\alpha})} \mathrm{d}t]$$

so that if, $|g(x,t)| \leq M$ the above integral gives us

$$\left| q\int\limits_{R}^{R_1} f(x,t)e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t \right|$$
$$\leq \frac{Mq}{q_0}\left\{ e^{-{R_1}^\alpha(\frac{1}{q^\alpha}-\frac{1}{{q_0}^\alpha})} + e^{-R^\alpha(\frac{1}{q^\alpha}-\frac{1}{{q_0}^\alpha})} - (\frac{1}{q}-\frac{1}{q_0})[e^{-{R_1}^\alpha \frac{1}{q^\alpha}} + e^{-R^\alpha \frac{1}{q^\alpha}} \right\}$$

hence by cauchy's criteria [8, 9] for uniform convergence on the given interval the integral

$$I = q\int\limits_{R}^{R_1} f(x,t)e^{-\frac{t^\alpha}{q^\alpha}}\mathrm{d}_\alpha t$$

converges uniformly on [q , ∞), if $\frac{1}{q} > \frac{1}{q_0} > 0$. □

Lemma 4.2. *If* $g(x,t) = q_0\int\limits_{0}^{x} f(u,t)e^{-\frac{u^\alpha}{{p_0}^\alpha}}\mathrm{d}_\alpha u$ *is bounded on [* p_0, ∞*) then the integral converges uniformly on [p ,* ∞*) for* $\frac{1}{p} > \frac{1}{p_0}$.

Proof. Consider, $0 < R < R_1$ and the integral;

$$p\int\limits_{R}^{R_1} f(u,t)e^{-\frac{u^\alpha}{p^\alpha}}\mathrm{d}_\alpha u = p\int\limits_{R}^{R_1} e^{-\frac{u^\alpha}{p^\alpha}} e^{-\frac{u^\alpha}{{p_0}^\alpha}} e^{\frac{u^\alpha}{{p_0}^\alpha}} f(u,t)\mathrm{d}_\alpha u \tag{4.2}$$

After rearranging the terms it gives

$$p\int\limits_{R}^{R_1} f(u,t)e^{-\frac{u^\alpha}{p^\alpha}}\mathrm{d}_\alpha u = \frac{p}{p_0}\int\limits_{R}^{R_1} e^{-x^\alpha(\frac{1}{p^\alpha}-\frac{1}{{p_0}^\alpha})} g_x(x,t)\mathrm{d}x$$

By using Fundamental Theorem of Calculus and applying integration by parts; it turns out to be

$$p\int\limits_{R}^{R_1} f(x,t)e^{-\frac{x^\alpha}{p^\alpha}}\mathrm{d}_\alpha x = \frac{p}{p_0}[g(R_1,t)e^{-{R_1}^\alpha(\frac{1}{p^\alpha}-\frac{1}{{p_0}^\alpha})} - g(R,t)e^{-R^\alpha(\frac{1}{p^\alpha}-\frac{1}{{p_0}^\alpha})}$$
$$+ (\frac{1}{p}-\frac{1}{p_0})\int\limits_{R}^{R_1} g(x,t)e^{-x^\alpha(\frac{1}{p^\alpha}-\frac{1}{{p_0}^\alpha})}\mathrm{d}x]$$

so that if, $|g(x,t)| \leq M$ the above integral gives us

$$\left| p \int_{R}^{R_1} f(x,t) e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x \right|$$
$$\leq \frac{Mq}{p_0} \left\{ e^{-R_1{}^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0{}^\alpha})} + e^{-R^\alpha(\frac{1}{p\alpha} - \frac{1}{p_0{}^\alpha})} - (\frac{1}{p} - \frac{1}{p_0})[e^{-R_1{}^\alpha \frac{1}{p\alpha}} + e^{-R^\alpha \frac{1}{p\alpha}} \right\}$$

hence by cauchy's criteria [8, 9] for uniform convergence on the given interval the integral

$$I = p \int_{R}^{R_1} f(x,t) e^{-\frac{x^\alpha}{p\alpha}} \mathrm{d}_\alpha x$$

converges uniformly on [p , ∞), if $\frac{1}{p} > \frac{1}{p_0} > 0$. □

Combining Lemma (4.2) and (4.3) the two-dimensional new integral transform of $f(x,t)$ converges uniformly on for $\frac{1}{p} > \frac{1}{p_0} > 0$ and $\frac{1}{q} > \frac{1}{q_0} > 0$, Hence the theorem. □

5 Conclusion

We have extend the one dimensional fractional Elzaki transformation towards the 2-– Dimensional fractional Elzaki Transformation and found out the condition for convergence and Uniform Convergence of this Transformation. Which will be useful to solve Non – homogeneous partial differential equations by using the relationship between 2 -– Dimensional Laplace and fractional Elzaki Transformations.

Acknowledgments: The author thanks to organizers of International e-Conference on Nonlinear Analysis and its Applications (2020) and Dr. A. P. Bhadane for his valuable guidance.

References

[1] A. Aghilli and H. Zeinali, *Joint Laplace – Fourier Transforms for Fractional PDEs*, New Trends in Mathematical Sciences, **2(3)** (2014), 166-177.

[2] A. Ansari, *The generalized Laplace Transform and fractional differential equations of distributed orders*, Differential Equations and Control Processes, **N3** (2012), 128-137.

[3] T. M. Elzaki and M. E. Salih, *On the new integral transform "Elzaki Transform" fundamental properties, investigations and applications*, Global Journal of Mathematical Sciences: Theory and Practical, **4(1)** (2011), 1-13.

[4] T. M. Elzaki and M. E. Salih, *The new integral transform "Tarig Transform" properties and applications to differential equations*, Elixir Appl. Math, **38** (2011), 4239–4242.

[5] M. Idrees, Z. Ahmed, M. Awais and Z. Perveen, *On the convergence of double Elzaki Transform*, International J. Advanced and Applied Sciences, **5(6)** (2011), 19-24.

[6] V. D. Gejji, *Fractional Calculus: Theory and Applications*, Narosa publication, 2014.

[7] S. D. Manjarekar and A. P. Bhadane, *Generalized Elzaki - Tarig Transformation and its applications to new fractional derivative with non – singular kernel*, **3(3)** PFDA, (2017), 1-6.

[8] S. D. Manjarekar and A. P. Bhadane, *New conformable fractional ELZAKI transformation: Theory and applications*, Malaya J. Matematik, vol. **S1** (2019), 619-625.

[9] O. Ozakan and A. Kurt, *The analytical solutions for conformable integral equations and integro–differential equations by conformable Laplace transform*, Opt. Quant. Electron, **50(81)** (2018).